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A CONSUL OF THE PHILOSOPHERS ON WHAT IS GIVEN AND WHAT IS NOT IN THE COSMOS

Fabio Acerbi

Résumé

Un texte byzantin qui applique le “langage des données” au cercles principaux sur la sphère céleste est ici présenté, édité, traduit et comparé avec sa source grecque. Dans son seul témoin manuscrit, le texte est attribué à un “consul des philosophes” ; des noms sont proposés pour cet auteur.

Mots clé: Astronomie byzantine, Mathématiques byzantines, consul des philosophes, langage des données.

Abstract

A Byzantine text, ascribed to a “consul of the philosophers”, applies the “language of the givens” to the main circles on the celestial sphere, showing which of them are “given”, and in what sense. This text is here presented, edited, translated, and compared with its source. A discussion about the author of the text is also provided.

Keywords: Byzantine astronomy, Byzantine mathematics, Consul of the philosophers, Language of the givens.

A CONSUL OF THE PHILOSOPHERS ON WHAT IS GIVEN AND WHAT IS NOT IN THE COSMOS

Fabio Acerbi

Introduction

A Byzantine text ("the Text" henceforth) appended to a witness of Aristotle's *Metaphysics* is here presented, edited, and translated.[1] The Text is interesting on several counts. First, it consistently and sustainedly uses the "language of the givens", a highly sectorial idiom whose founding exposition is Euclid's *Data* and which is infrequently met in extant Greek mathematical works.[2] Byzantine texts of this kind are exceedingly rare.[3] The Text

* The perceptive remarks of one of the referees have improved my argument.

1 The existence of the Text was first reported in A. Tihon, "Les sciences exactes à Byzance", *Byzantion* 79 (2009), 380–434: 399 n. 55.

2 Euclid's *Data* are edited in vol. VI of J.L. Heiberg, H. Menge, *Euclidis opera omnia*, I–VIII, Lipsiae 1883–1916; see also the study Ch.M.Taisbak, ΔΕΔΟΜΕΝΑ. *Euclid's* Data *or the Importance of Being Given* (Acta Historica Scientiarum Naturalium et Medicinalium 45), Copenhagen 2003. For the "language of the givens" see most recently F. Acerbi, *The Logical Syntax of Greek Mathematics* (Sources and Studies in the History of Mathematics and the Physical Sciences), Heidelberg – New York 2021, sect. 2.4. This stylistic resource was mainly applied in the so-called "analytical *corpus*", now almost entirely lost; the standard account of the analytical *corpus* is Pappus, *Collectio* VII.

3 In the Byzantine technical *corpus*, the only structured exposition framed in the "language of the givens" I know of is Book VI of Barlaam's *Logistic*, edited in P. Carelos, *Βαρλαὰμ τοῦ Καλαβροῦ, Λογιστική. Barlaam von Seminara, Logistiké* (Corpus philosophorum Medii Ævi. Philosophi byzantini 8), Athens – Paris – Bruxelles 1996, 94–113. For Latin West, see

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applies the "language of the givens" to the main circles on the celestial sphere, showing which of them are "given", and in what sense. Second, the Text is less an exercise in checking whether assigned mathematical objects satisfy exotic definitions than an exercise in the logic of complex predicates. Third, the style of the Text is very characteristic, for its author—partly spurred by the subject-matter—exhibits such a fondness for conjunctions formulated by τε … καί correlatives as I have never found in other Byzantine scientific works. This stylistic feature might be useful, because, Fourth, the Text is ascribed to a "consul of the philosophers" whose name can only be guessed, even if, as we shall see, some natural candidates are at hand. In keeping with this ascription, the Text has a marked pedagogic ring; it might be the redaction of lecture notes. The Text is virtually free of copying mistakes, suggesting that it was copied first-hand. Fifth, the Text *does not* contain gross misunderstandings of the "language of the givens", something I regarded as surprising in the first place—granted, it *does* contain some quirks, but we shall see that the matter is not entirely trivial. Sixth, my surprise before the absence of gross misunderstandings vanished when I found the Greek source ("the Source" henceforth) of the Text.

In the following sections, I shall first present the Text and its only manuscript witness, then give its Greek text and an annotated translation. Finally, I shall give the Greek text and a translation of the Source, compare it with the Text, and review the candidates for our "consul of the philosophers".

Introducing the Text

The Text applies the "language of the givens" to the main circles on the celestial sphere, showing which of them are "given", and in what sense. The relevant circles are listed at the beginning of the Text. These are first the so-called latitudinal circles: the equator; the two tropics, that is, the two circles parallel to the equator that are tangent to the ecliptic; the Arctic and Antarctic circles, namely, the two circles parallel to the equator that are tangent to the horizon, that is, the circles that, at an assigned latitude, delimit the portion of the celestial sphere which never set (resp. rise). Other relevant circles (which the Text also calls "latitudinal") are the horizon, the meridian—which is the circle that passes

Jordanus de Nemore's *De numeris datis*, edited in B.B. Hughes, *Jordanus de Nemore*, *De numeris datis* (Publications of the Center for Medieval and Renaissance Studies, UCLA 14), Berkeley – Los Angeles – London 1981.

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through the north pole and the local zenith, and which the Sun crosses at midday—and the colures, which pass through the north pole and either the equinoctial or the solstitial points.[4] The ecliptic itself—that is, the yearly trajectory of the Sun—is a circle whose partitions (the zodiacal signs) divide the sphere in longitude. Finally, there are the circles of the Moon and of the planets. Some of these circles are great circles on the celestial sphere: these are the equator, the meridian, the colures, the ecliptic, and the horizon. The two tropics and the Arctic and Antarctic circles are not great circles; moreover, the size and the position of the latter two depend on the observer's position, very much as the position of the horizon does. As is clear, then, some of these circles are intrinsic features of the celestial sphere, and as such they enjoy specifically invariant properties "in themselves"; other circles only enjoy their characteristic properties "with respect to us".

What does it mean that some of these circles are "given in position" or "given in magnitude"? Roughly speaking, it means that such circles always hold the same place, or that their size is well-defined, respectively. To see how these notions were formalized, let us read in parallel *Data* def. 4 and 1 (in this order) and the corresponding definitions that open the Text:[5]

Data	the Text
τῇ θέσει δεδόσθαι λέγονται σημεῖά τε καὶ γραμμαὶ καὶ γωνίαι ἃ τὸν αὐτὸν ἀεὶ τόπον ἐπέχει	δεδόσθαι λέγεται σημεῖον τὲ καὶ γραμμὴ ἃ τὸν αὐτὸν ἔχει ἀεὶ τόπον
both points and lines and angles that always hold the same place are said to be given in position	both a point and a line that always hold the same place are said to be given
δεδομένα τῷ μεγέθει λέγεται χωρία τε καὶ γραμμαὶ καὶ γωνίαι οἷς δυνάμεθα ἴσα πορίσασθαι	μεγέθη δὲ δεδόσθαι λέγεται οἷς δυνάμεθα ἴσα πορίσασθαι
both regions and lines and angles for which we can procure equals are said to be given in magnitude	magnitudes for which we can procure equals are said to be given

In the *Data*, the intuitive notion of "place" is retained—accordingly, the Euclidean space is an absolute space—while "size" is replaced by the relational property of equality, which is as it were "saturated" by a magnitude we are able to "procure"; the result is the

4 As we shall see, "colure" can also be used as a generic name.

5 For the *Data*, see Heiberg, Menge, *Euclidis Opera Omnia* [cit. n. 2], VI, 2.9–10 and 2.2–3, respectively. The scarce variant readings in the text of the *Data* do not identify a specific line of tradition as the one followed by the Text.

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predicate "being given in magnitude". In the *Data*, as elsewhere in Greek geometry, "to procure" means being able to show, by means of an argument or of a construction, that there is indeed such an "equal" magnitude.[6] Comparing the Euclidean definitions and those that open the Text, a first quirk is apparent: the Text does not define "being given in position" and "being given in magnitude", but defines "being given" for points and lines first, and then the same notion for generic magnitudes. In so doing, the Text somehow mixes up the qualifiers "in position" and "in magnitude", which specify the generic relation "being given", and the objects that fall under the range of the resulting predicates (the "magnitudes" in the second definition replace the specific geometric objects listed in *Data* def. 1). However, in what follows the Text keeps faithful to the original definitions, for it always refers to "being given in position" and "being given in magnitude".

As we have seen above, and as the second paragraph of the Text explains, all the circles on the celestial sphere that are great circles are equal to one another. Consequently, they are given in magnitude according to *Data* def. 1. The tropics are also given in magnitude, for they are circles on the surface of the sphere that are placed at a well-defined distance from the equator. As there always are two twin circles at a well-defined distance from the equator, and such circles are equal to one another by symmetry, they are also "given in magnitude" according to *Data* def. 1. The reader might object that *any* circle on the surface of a sphere is placed at a well-defined distance from a suitable great circle and has a twin circle placed at the same distance. Will *any* circle on the surface of a sphere be given too? Yes, because the reader, by referring to a *well-defined* distance of any circle, necessarily means any *assigned* circle on the surface of an *assigned* sphere: the definitions of the *Data* give a mathematical meaning to the intuitive notion "being assigned". This leads us to the main subtlety involved in the argument expounded in the Text. The point is that the celestial sphere is a second-order mathematical fiction: it is not a "real" mathematical sphere whose diameter is fixed, but a scale-invariant sphere which is used to compute angular distances only. In the *Almagest*, Ptolemy assigns a notional size to this sphere (or, better said, to any of its great circles), taking for instance its diameter to be 120 linear "degrees".[7] Consequently, and in a mathematically impeccable way, the celestial sphere

6 The equal magnitude may also be "procured" in a definition: a case in point is the definition of a right angle in *Elem*. I.def.10. A right angle is given in magnitude for this reason.

7 See J.L. Heiberg, *Claudii Ptolemaei opera quae exstant omnia*, I.1–2, *Syntaxis Mathematica*, Lipsiae 1898–1903, I.1, 77.6–13.

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is “given in magnitude”, and any of the circles on it whose size is well-defined—most obviously, any great circle—is “given in magnitude” both intuitively and, for non-great circles, via the trick of the twin circles. Conversely, there are circles on the surface of the celestial sphere that are not given in magnitude: these are the Arctic and the Antarctic circle, whose size depends on the elevation of the (north) pole above the horizon, that is, on the observer’s position in latitude. The “circles” of the Moon and of the planets are not given in magnitude either, because, as the Text asserts, “we cannot procure a circle equal to each of them”—actually, the trajectories traced by the planets on the surface of the sphere are not circles at all.

The issue of “being given in position” is even trickier, and the Text is forced to introduce a distinction that, while at home in philosophical arguments, is outlandish by mathematical standards. There are circles on the surface of the sphere that are obviously given in position: these are the equator and the tropics, for the daily motion of the heavens makes each of them rotate onto itself. Conversely, there are circles on the surface of the sphere that are obviously not given in position: these are the Arctic and the Antarctic circle, whose position depends, as said, on the observer’s position in latitude. The positions of the meridian and of the horizon also depend on the observer’s position; accordingly, these circles are not given in position. The status of the zodiac is more problematic, for on the one hand the daily motion of the heavens carries it around in the sky, but on the other hand—disregarding the precession of the equinoxes—its position is determined by the fixed stars. The Text solves this aporia in a typically philosophical way, namely, by introducing a linguistic distinction: the zodiac “is given in itself but is not given with respect to us”. A standard exegetic tool—an aporia raised by a seemingly paradoxical state of affairs—also operates in the discussion of the colures. For as the meridian and the colures pass through the poles of the equator, they go the one onto the other during the daily motion. Thus, the meridian and the colures are one and the same thing. Moreover, all of them are curtailed circles, for the horizon makes half of them invisible (this is true of any other great circle on the sphere, by the way). Accordingly, the meridian counts as two different objects depending on the observer being placed in a given location or in its antipode. These facts are upgraded to an aporia by the Text, which wonders “how come the one becomes two, and how come sometimes it is called ‘meridian’ sometimes it is named ‘colure’”.

As any of the two species of the predicate “to be given” may hold or not hold of the circles on the surface of the celestial sphere, four combinations are possible in con-

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junction, namely, being given "both in position and in magnitude", "neither in position nor in magnitude", "in position but not in magnitude", and "in magnitude but not in position". The Text lists all of these combinations, along with the circles to which each of these complex predicates applies. I surmise that setting forth such a fourfold partition was the main goal of the Text.[8]

The Text also provides pieces of astronomical information not strictly related to the main subject-matter; the latter, of course, also includes a definition of the species of the "givens" and of all circles involved, as well as some relevant properties of these circles. The additional pieces of information are the varying inclination of the ecliptic with respect to the equator, a sketchy outline of the motions of the Sun, the Moon, and the planets, and accordingly a preliminary clarification of the terms "behind-leaving" (ὑπολειπτικός) and "forward-carrying" (προηγητικός)—the latter a synonym of "retrogradation"—which describe the direction of the motion of the heavenly bodies.[9]

The Text does not contribute any new mathematical results. This academic exercise makes sense only as the redaction of a lecture, perhaps a well-thought introduction—embedded in a logical framework, and made lively by a couple of aporias—to the system of circles on the celestial sphere. This framework explains the points of terminology, the presence of the additional notions, the several cross-references, the insistence on the logic of complex predicates, the exegetic tricks, which are fully justified only before an audience. The use of the "language of the givens" points to extensive readings of the *Almagest*, in whose Books I and III this stylistic resource is repeatedly deployed in its full mathematical import.[10] The argument of a course including such

[8] One is reminded of a similar exercise in the logic of complex predicates in Marinus' *prolegomena* to the *Data*: see the discussion in F. Acerbi, *Euclide*, *Tutte le Opere*, Milano 2007, 2487–2499. Readers unfamiliar with Italian can find a similar discussion in M. Sialaros, R. Matera, J. Gerhold, G. Gamarra Jordán, "Searching for Definitions: Marinus' *Introduction to Euclid's Data*", *SCIAMVS* 20 (2019), 119–155.

[9] Translating the Greek terminology is tricky: see G.J. Toomer, *Ptolemy's Almagest*, London 1984, 20.

[10] The involved chapters are *Almagest* I.10 (in Heiberg, *Claudii Ptolemaei opera* [cit. n. 7], I.1, 37.20–42.6), I.13 (*ibid*., 71.14–72.10, 73.11–74.8), III.5 (*ibid*., 242.14–243.15, 245.5–246.5, 247.15–248.19, 250.8–251.9). I have not found anything similar to the Text in the scholia to the *Almagest* or to the *Data*.

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an introductory lecture is a matter of speculation (astronomy? elementary logic?) and I shall not indulge in it. The relation of the Text with the Source will be investigated in the final section of this paper. Before doing that, let us read both.

The Manuscript Witness of the Text

The Text is uniquely witnessed in ff. 252r–258r of the manuscript Città del Vaticano, Biblioteca Apostolica Vaticana, gr. 255 (*Diktyon* 66886), a stemmatically independent copy of Aristotle's *Metaphysics*.[11] Vat. gr. 255 is a paper manuscript whose watermarks—all of them are alphabet letters—point to the first half of the 14$^{\text{th}}$ century. This is confirmed by the script, which Daniele Bianconi has kindly dated, on my request, to the central decades of the same century as for our text, to the first quarter of the same century as for the *Metaphysics*. Vat. gr. 255 is made of 3 recent folios + 3 folios, numbered 1 and 1$^{\text{a–b}}$, added a little after the date of copy + 256 folios (= 32 quaternions), numbered 1$^{\text{c–d}}$, 2–251, 251$^{\text{a–d}}$ + 7 folios at the end, now bound in disorder (the correct order is 256, 257, 252–255, 258). Folios 1r, 1$^{\text{b}}$v, 1$^{\text{c}}$v, 251$^{\text{a–d}}$ are blank; f. 258v contains only a later six-word inscription.

Aristotle's *Metaphysics*—which begins on the recto of f. 1$^{\text{d}}$, after a blank folio—was copied from a defective exemplar: spaces for unreadable words are frequently left in the text. Two long passages that were omitted by the main copyist are restored by different, and slightly later, hands, on ff. 1v–1$^{\text{b}}$r and 251r–v. The two correctors used identical catching phrases to refer to the location of their integrations: they likely worked in collaboration. A different, and a bit later, hand copied again, on f. 1$^{\text{c}}$r, the beginning of the *Metaphysics*.

The Text, which is written by a hand different from any of the above, and which bears no relation with any of the arguments expounded in Aristotle's treatise, was appended to it, as shown by the quire structure of Vat. gr. 255.

[11] See the description in G. Mercati, P. Franchi de' Cavalieri, *Codices Vaticani graeci. Codices 1–329*, Romae 1923, 333–334. The manuscript is accessible online at https://digi.vatlib.it/view/MSS_Vat.gr.255. For the stemmatic position of Vat. gr. 255 (*siglum* V$^{\text{d}}$), see D. Harlfinger, "Zur Überlieferungsgeschichte der *Metaphysik*", in P. Aubenque (ed.), *Études sur la Métaphysique d'Aristote. Actes du VI$^{\text{e}}$ Symposium Aristotelicum*, Paris 1979, 7–36: 20 and 27 (stemma). All subsequent scholarship adopts Harlfinger's stemma as regards the position of V$^{\text{d}}$.

It is likely that Vat. gr. 255 was present in the Vatican Library since its early years, as shown by the early inventories. Matters are complicated by the fact that Vat. gr. 257 (15th c.; *Diktyon* 66888) also contains the *Metaphysics* and nothing else. For this reason, two of the reference editions of the early Vatican inventories do not agree in their identifications of the item recorded in Vigili's catalogue of 1508–10.[12] However, it is almost certain that both manuscripts were recorded in some inventories, and as early as 1481.[13] Thanks to the presence of the *dictio probatoria*, the only unambiguous identifications relate Vat. gr. 255 and Vat. gr. 257 to specific items of the 1533 inventory.[14]

Edition of the Text

I have regularized punctuation and accents, with the sole exception of the enclitics, and introduced a segmentation of the Text. Sequences that correspond to statements we shall also find in the Source are underlined.

12 Compare R. Devreesse, *Le fonds grec de la Bibliothèque Vaticane des origines à Paul V* (Studi e Testi 244), Città del Vaticano 1965, 57 (this is the earliest occurrence, in 1475), 108, 142, 161, 219, 245, 311 (Vat. gr. 255) and 92, 130, 198, 245, 281, 326, 406 (Vat. gr. 257), with G. Cardinali, *Inventari di manoscritti greci della Biblioteca Vaticana sotto il pontificato di Giulio II (1503-1513)* (Studi e Testi 491), Città del Vaticano 2015, 181, 297 (Vat. gr. 257). Apparently, Vat. gr. 255 had a *paonatio*/black binding, Vat. gr. 257 a red one.

13 See Devreesse, *Le fonds* [cit. n. 12], 92, 142, 219, 245, 311 (Vat. gr. 255) and 108, 130, 198, 245, 281 (Vat. gr. 257), and the new editions of the 1518 inventory, M.L. Sosower, D.F. Jackson, A. Manfredi, *Index seu inventarium Bibliothecae Vaticanae divi Leonis pontificis optimi : anno 1518 c. Series graeca* (Studi e Testi 427), Città del Vaticano 2006, 87 nr. [670] (Vat. gr. 255) and 35 nr. [263] (Vat. gr. 257), and of the 1533 inventory, M.R. Dilts, M.L. Sosower, A. Manfredi, *Librorum Graecorum Bibliothecae Vaticanae Index a Nicolao De Maioranis compositus et Fausto Saboeo collatus Anno 1533* (Studi e Testi 384), Città del Vaticano 1998, 36 nr. 274 (Vat. gr. 257), and 100 nr. 851 (Vat. gr. 255, but the item is left unidentified in this edition).

14 See Devreesse, *Le fonds* [cit. n. 12], 311, and Dilts, Sosower, Manfredi, *Librorum Graecorum* [cit. n. 13], 100. The word γένος recorded in the inventory is the last of f. 1cr; as for Vat. gr. 257, the *dictio probatoria* is ἐνδέχεται on f. 3r. Note that the item where Vat. gr. 255 is described qualifies it as *sine tabulis*, whereas all other inventories record the presence of a binding. For the *dictio probatoria*, see D. Williman, K. Corsano, "Tracing Provenances by *Dictio Probatoria*", *Scriptorium* 53 (1999), 124–145, and Dilts, Sosower, Manfredi, *Librorum Graecorum* [cit. n. 13], ix–xviii.

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$|_{256r}$ τοῦ ὑπάτου τῶν φιλοσόφων

<**1**> δεδόσθαι λέγεται σημεῖον τὲ καὶ γραμμὴ ἃ τὸν αὐτὸν ἔχει ἀεὶ τόπον· μεγέθη δὲ δεδόσθαι λέγεται οἷς δυνάμεθα ἴσα πορίσασθαι. καὶ ὅπως μὲν ἕκαστον τούτων ἔχει, ἐφεξῆς θεωρητέον· χρὴ δὲ γινώσκειν ὡς παρὰ μὲν τῷ γεωμέτρῃ τὰ δεδομένα ἐξ ἀφαιρέσεως ἀποδείκνυται καὶ καθολικώτερον καὶ ἁπλούστερον. ἐπὶ μέντοι ἀστρονομίας οὐκ ἐκτὸς ταῦτα θεωρεῖται τῆς ὕλης· διὸ καὶ σαφεστέρα ἐνταῦθα ἡ τῶν τοιούτων δεδομένων κατανόησις. φαμὲν γὰρ τὴν οὐρανίαν σφαῖραν κατὰ μὲν πλάτος εἰς πέντε διαιρεῖσθαι παραλλήλους κύκλους – δύο μὲν τοὺς ὁρίζοντας τά τε ἀειφανῆ καὶ τὰ ἀφανῆ, δύο δὲ ἑτέρους τόν τε θερινὸν τροπικὸν καὶ τὸν χειμερινὸν τροπικόν, μεταξὺ δὲ τούτων τὸν ἰσημερινόν – κατὰ δὲ μῆκος τοῖς δώδεκα λεγομένοις ζῳδίοις – ἅπερ ἀλληλουχούμενα καὶ ἑνούμενα τὸν ζῳδιακὸν καὶ λοξὸν ἀποτελεῖ κύκλον – κατὰ δὲ βάθος τοῖς τῶν ἑπτὰ πλανήτων διαιρεῖται κύκλοις ἡ σφαῖρα.

<**2**> $|_{256v}$ εἰσὶ δὲ καὶ ἕτεροι κύκλοι, ὅ τε ὁρίζων ὁ μεσημβρινὸς καὶ οἱ κόλουροι, κατὰ πλάτος καὶ αὐτοὶ δοκοῦντες διαιρεῖν τὸ οὐράνιον σῶμα. τούτων ὅσοι μὲν δίχα διαιροῦσι τὴν σφαῖραν μέγιστοι τῶν ἄλλων εἰσὶ καὶ ἀλλήλοις ἴσοι· ὥσπερ γὰρ ἐπὶ τοῦ κύκλου ἡ διὰ τοῦ κέντρου ἠγμένη εὐθεῖα, ἣν καὶ διάμετρον λέγομεν, μεγίστη ἐστὶ τῶν παρ᾽ ἑκάτερα εὐθειῶν, οὕτω καὶ ἐπὶ τῆς σφαίρας ὁ δίχα διαιρῶν κύκλος τὴν σφαῖραν μέγιστος τῶν παρ᾽ ἑκάτερα κύκλων ἐστίν. εἰσὶ δὲ οὗτοι ὅ τε ὁρίζων ὁ ἰσημερινός, ὁ μεσημβρινὸς καὶ ὁ ζῳδιακός· πάντες γὰρ οὗτοι δίχα τὴν σφαῖραν τέμνουσιν καὶ εἰσὶν ἴσοι.

<**3**> ἀλλ᾽ ὁ μὲν ἰσημερινὸς καὶ θέσει καὶ μεγέθει δέδοται, θέσει μὲν ὅτι τὸν αὐτὸν ἀεὶ τόπον κατέχει· εἴρηται γὰρ ἐν τοῖς ὅροις ὅτι σημεῖα καὶ γραμμαὶ δεδόσθαι λέγεται τὰ τὸν αὐτὸν ἀεὶ τόπον ἔχοντα, γραμμὴ δὲ καὶ ὁ κύκλος, καὶ ἔνθα γραμμή, καὶ σημεῖον, οὐ μὴν τὸ ἀνάπαλιν. πῶς δὲ ὁ ἰσημερινὸς τὸν αὐτὸν ἀεὶ τόπον ἔχει (ἡ γὰρ θέσις ἐν τόπῳ); ὅτι διὰ τοῦ Κριοῦ καὶ τοῦ Ζυγοῦ δίεισιν οὗτος, Κριὸς δὲ καὶ Ζυγὸς $|_{257r}$ ἀμετάπτωτα καὶ τὸν αὐτὸν ἀεὶ τόπον κατέχοντα. οἱονεὶ γὰρ πέπηγεν ἐν τῷ οὐρανῷ τὰ ζῴδια πάντα, καὶ ἔστιν ἀμετακίνητα, ὥστε εἰ ἐν τοῖς ἀμετακινήτοις τῶν ἄστρων θεωρεῖται ὁ ἰσημερινός, καὶ οὗτος ἀμετακίνητός ἐστι καὶ τὸν αὐτὸν ἀεὶ τόπον ἔχει, καὶ τῇ θέσει δέδοται. δέδοται δὲ καὶ τῷ μεγέθει, ὡς εἴρηται· ἔστι γὰρ αὐτῷ πορίσασθαι ἴσον, οὐ μόνον τὸν ὁρίζοντα, ἀλλὰ καὶ τὸν μεσημβρινὸν καὶ τὸν ζῳδιακόν – ἢ τοῦ ζῳδιακοῦ τὸ μεσαίτατον· οἱ μὲν γὰρ ἄλλοι κύκλοι γραμμικοὶ καὶ ἀπλατεῖς ἐπινενόηνται, ὁ δὲ ζῳδιακὸς καὶ πλάτος ἔχειν ἀξιόλογον λέγεται, ὥστε ὁ μεσαίτατος ἐν αὐτῷ κύκλος μόνος εἰς δύο τέμνων τὴν σφαῖραν εὑρίσκεται.

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<**4**> ἀλλὰ δὴ καὶ οἱ παρ᾽ ἑκάτερα αὐτοῦ ὅ τε θερινὸς τροπικὸς καὶ ὁ χειμερινὸς τροπικὸς καὶ τῇ θέσει καὶ τῷ μεγέθει δέδονται, τῇ μὲν θέσει ὅτι οὐδέτερος αὐτῶν τὸν ζῳδιακὸν ὑπερβαίνει· ὁ μὲν γὰρ κατὰ τὴν ἀρχὴν τοῦ Καρκίνου καὶ οἱονεὶ τὸ πρῶτον λεπτὸν ἢ σημεῖον λαμβάνεται, μεθ᾽ ἣν ἀρχὴν ἢ μεθ᾽ ὃ πρῶτον λεπτόν – καθ᾽ ὃ |257v καὶ ἡ μεγίστη πασῶν ἡμερῶν γίνεται – τὴν θερινὴν τροπὴν συμβέβηκε γίνεσθαι. εἰ δ᾽ ὁ ζῳδιακὸς καὶ ὁ ἐν τούτῳ Καρκίνος καὶ ἡ τοῦ Καρκίνου ἀρχὴ πάντως ἀμετακίνητα καὶ τὸν αὐτὸν ἀεὶ τόπον ἔχοντα, καὶ ὁ θερινὸς τροπικὸς ἄρα τῷ τόπῳ τὲ καὶ τῇ ἐν αὐτῷ θέσει δέδοται. οὕτω δὲ καὶ ὁ χειμερινὸς τροπικὸς δέδοται τῷ τόπῳ τὲ καὶ τῇ θέσει· εἰς γὰρ τὸ πρῶτον τοῦ Αἰγόκερω λεπτὸν ἐπινενόηταί τε καὶ γίνεται. ὡσαύτως δὲ καὶ τῷ μεγέθει οὗτοι οἱ δύο κύκλοι δέδονται· τῷ τε γὰρ θερινῷ τροπικῷ ἔστιν ἴσον πορίσασθαι τὸν χειμερινὸν τροπικόν, καὶ τῷ χειμερινῷ τροπικῷ τὸν θερινὸν τροπικὸν ἔμπαλιν· ὧν γὰρ ἡ πρὸς τὸν ζῳδιακὸν διάστασις ἴση, καὶ αὐτοὶ ἴσοι. ἀλλ᾽ οὗτοι μὲν οἱ τρεῖς, ὅ τε ἰσημερινὸς ὁ θερινὸς τροπικὸς καὶ ὁ χειμερινὸς τροπικὸς τῇ τε θέσει τῷ τε μεγέθει δέδονται, ὥσπερ εἴρηται.

<**5**> ἐκ τοῦ ἐναντίου δὲ ὁ ἀρκτικὸς καὶ ἀνταρκτικός, τουτέστιν ὅ τε τὰ ἀειφανῆ ὁρίζων καὶ ὁ τὰ ἀφανῆ, οὔτε τῇ θέσει οὔτε τῷ μέγεθει δέδονται. ἔστι δ᾽ ὅτε καὶ παντελῶς οὐκ εἰσίν, οὔθ᾽ ὁ μὲν |252r ἀφανὴς ὢν οὔτε ἀειφανὴς ὁ ἕτερος· ἐν γὰρ τῇ διακεκαυμένῃ καὶ ἀοικήτῳ διὰ τῶν πόλων ἑκατέρων δίεισιν ὁ ὁρίζων, καὶ πάντα τὰ ἄστρα τά τε βορειότερα καὶ τὰ νοτιώτερα καὶ ἀνατέλλουσι καὶ δύνουσιν, ὥστε οὔτε τὰ βορειότερα ἀειφανῆ εἰσὶν οὔτε τὰ νοτιώτερα ἀφανῆ. εἰς δὲ τὰς παρ᾽ ἑκάτερα ἐγκλίσεις τῆς γῆς, παρ᾽ οἷς μὲν βορειότατοί τε καὶ νοτιώτατοί εἰσι, εἰς μὲν τὴν ἡμετέραν εὔκρατον βορειότατος ὁ ἀειφανὴς μικρὸν ἔχων τὸν πόλον ἐξαίροντα καὶ μικρὸν τὸν τοῦ ἀειφανοῦς κύκλον γράφων, ὁ δὲ ἀφανὴς νοτιώτατος μικρὸν ὑπεράνω τοῦ πόλου τὸν ὁρίζοντα ἔχων ἐξαίροντα καὶ μικρὸν γράφων τὸν τῶν ἀφανῶν κύκλον. ἐπεὶ δὲ εἰς τὴν καθ᾽ ἡμᾶς εὔκρατον ἑπτὰ παραλλήλοις κλίμασι τὸ τῆς γῆς ἔγκλιμα τέμνεται, καθ᾽ ἕκαστον κλίμα, ὡς ἐπὶ τὸ βορειότερον ἀρχομένων ἡμῶν ὁ πόλος τὲ πλεῖον ἐξαίρειται ὁ βόρειος καὶ ὁ κύκλος τῶν ἀειφανῶν μείζων γίνεται. ὡσαύτως δὲ ὁ νότιος πόλος πλεῖον καταβιβάζεται καὶ ὁ ὁρίζων ὑψηλότερος γίνεται, ἀναλόγως δὲ καὶ ὁ τὰ ἀφανῆ γράφων κύκλος μείζων γίνεται. καθ᾽ ἑαυτοὺς μὲν οὖν ὅ τε ἀρκτικὸς καὶ ὁ ἀνταρκτικὸς οὔτε τῇ θέσει οὔτε |252v τῷ μεγέθει δέδονται – πρὸς γὰρ τὰς διαφόρους οἰκήσεις καὶ τὰ διάφορα τῆς γῆς κλίματα τήν τε θέσιν καὶ τὸ μέγεθος ὑπαλλάττουσι – πρὸς ἀλλήλους δὲ οὔτε τὴν θέσιν οὔτε τὸ μέγεθος ἀλλοιοῦσιν, ἀλλ᾽ οἱονεὶ καὶ κατ᾽ ἀμφότερα δέδονται· ὅτε γὰρ ὁ τῶν ἀειφανῶν κύκλος βορειότατός ἐστι καὶ σμικρότατος, τηνικαῦτα καὶ ὁ τῶν ἀφανῶν κύκλος νοτιώτατός ἐστι καὶ σμικρότατος, ὅτε δὲ ἐκ τοῦ ἐναντίου ὁ τῶν

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ἀειφανῶν μέγιστος, καὶ ὁ τῶν ἀφανῶν μέγιστος καὶ τῇ τε θέσει ὅμοιος καὶ τῷ μεγέθει ἴσος, ἐν τῷ μεταξὺ δὲ τῶν τε σμικροτάτων καὶ τῶν μεγίστων κατὰ τὸ ἀνάλογον, ὥστε καθ᾽ ἑαυτοὺς μὲν οὗτοι οἱ δύο κύκλοι ὁ ἀρκτικὸς καὶ ὁ ἀνταρκτικὸς οὔτε τῇ θέσει οὔτε τῷ μεγέθει δέδονται, πρὸς ἀλλήλους δὲ τῇ τε θέσει καὶ τῷ μεγέθει δέδονται. καὶ περὶ μὲν τῶν παραλλήλων ταῦτα.

<**6**> ὁ δὲ ζῳδιακὸς καὶ αὐτὸς τῇ τε θέσει καθ᾽ ἑαυτὸν καὶ τῷ μεγέθει δέδοται· τόν τε γὰρ τόπον τὸν αὐτὸν εὑρίσκεται κατέχων ἀεί, καὶ ἔστιν ἡμῖν πορίσασθαι αὐτῷ ἴσον ἕτερον, τόν τε ἰσημερινὸν τὸν μεσημβρινὸν καὶ τὸν ὁρίζοντα· πάντες γὰρ οὗτοι διχοτομοῦσι |253r τὴν σφαῖραν. εἴρηται δὲ "καθ᾽ ἑαυτόν" ὅτι ὡς πρὸς ἡμᾶς οὐ τηρεῖ τὴν αὐτὴν ἀεὶ θέσιν· ἔστι γὰρ λοξὸς οὐ παράλληλος, καὶ παρ᾽ οἷς μέν ἐστι βορειότερος παρ᾽ οἷς δὲ νοτιώτερος, καὶ ὡς πρὸς τὴν ὀρθὴν σφαῖραν παρ᾽ οἷς μὲν πλαγιώτερος παρ᾽ οἷς δὲ ὀρθώτερος, τουτέστι παρ᾽ οἷς μὲν ὀξυγωνιώτερος παρ᾽ οἷς δὲ ὀρθογωνιώτερος παρ᾽ οἷς δὲ ἀμβλυγωνιώτερος. ἐπεὶ γάρ ἐστι σημεῖον τοῦ ὀρθὴν εἶναι τὴν σφαῖραν ὁ ἰσημερινὸς τέμνεται δὲ ὁ ἰσημερινὸς τῷ ζῳδιακῷ, ἐγγὺς μὲν τῆς τομῆς ὀξυγωνιωτέρα ἐστὶν ἡ λόξωσις, ἀπωτάτω δὲ τῆς τομῆς οἱονεὶ ἀμβλυγώνιος, μεταξὺ δὲ τούτων οἱονεὶ ὀρθωγώνιος. οὕτω μὲν οὖν καὶ περὶ τοῦ ζῳδιακοῦ, ὃς καὶ κατὰ μῆκος τέμνει τὴν σφαῖραν, ὡς εἴρηται.

<**7**> ἐπεὶ δὲ τὸ πλῆρες τῆς διαιρέσεως οὔτε τὸ ἐξ ἀμφοτέρων καταφατικὸν ἀπαιτεῖ μόνον, οἷον τὸ καὶ τῇ θέσει καὶ τῷ μεγέθει δεδόσθαι, οὔτε δὲ τὸ ἐξ ἀμφοτέρων ἀποφατικὸν μόνον, οἷον τὸ μήτε τῇ θέσει μήτε τῷ μεγέθει δεδόσθαι, ἀλλὰ καὶ τὸ ἐξ ἀμφοτέρων συμμιγές, οἷον θέσει μὲν δεδόσθαι μεγέθει δ᾽ οὔ, ἢ θέσει μὲν μὴ[15] δεδόσθαι μεγέθει δέ, ἴδωμεν καὶ ταῦτα παρ᾽ οἵοις τῶν οὐρανίων |253v εὑρίσκεται. ὁ μὲν οὖν μεσημβρινὸς καὶ ὁ ὁρίζων τῇ θέσει μὲν οὐ δέδονται τῷ μεγέθει δὲ δέδονται· ὁ μὲν γὰρ τόπος αὐτοῖς παρὰ τὰς διαφόρους οἰκήσεις ὑπήλλακται· οὐ γάρ ἐστιν ἡ γῆ οὐδὲ ἡ ταύτης ἐπιφάνεια ὁμαλὴς καὶ ἐπίπεδος ἀλλὰ σφαιρικὴ καὶ ἀνώμαλος[16]. διὰ τοῦτο οὔτε παρὰ πᾶσιν ὁ αὐτὸς ὁρίζων, ἀλλὰ παρ᾽ οἷς μὲν ταπεινότερος παρ᾽ οἷς δὲ ὑψηλότερος, οὔθ᾽ ὁ μεσημβρινὸς ὁ αὐτὸς παρὰ πᾶσιν· οὔτε γὰρ ἡ ἀνατολὴ καὶ ἡ δύσις ἡ αὐτὴ παρὰ πᾶσι διὰ τὰ μεσολαβοῦντα τῆς γῆς κυρτώματα καὶ ἐγκλίματα, οὔτε μεσουράνημα τὸ αὐτό· τὸ γὰρ τοῦ ἡλίου μεσουράνημα τὸν μεσημβρινὸν δείκνυσιν ὑπὲρ κεφαλῆς ἡμῶν τηνικαῦτα τοῦ ἡλίου ὄντος. τῷ μεγέθει δὲ καὶ οὗτοι δέδονται· τῷ τε γὰρ ὁρίζοντι ἔστιν

15 μὴ s.l.
16 ἀνόμαλος cod.

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ἴσον πορίσασθαι τὸν μεσημβρινόν, καὶ τῷ μεσημβρινῷ τὸν ὁρίζοντα. ταῦτα καὶ περὶ τούτων.

<**8**> ἐφεξῆς δὲ περὶ τῶν κατὰ βάθος ἴδωμεν κύκλων. εἰσὶ δὲ οὗτοι ἑπτά, ὁ τοῦ Κρόνου, ὁ τοῦ Διός, ὁ τοῦ Ἄρεως, ὁ τοῦ ἡλίου, ὁ τῆς Ἀφροδίτης, ὁ τοῦ Ἑρμοῦ καὶ ὁ τῆς σελήνης. καὶ πάντες μὲν οἱ |254r ῥηθέντες ἀστέρες ἰδίαν καὶ προαιρετικὴν κινοῦνται κίνησιν, ἀλλ᾽ οὐ πάντες ὁμοίως. ἥλιος μὲν γὰρ καὶ σελήνη ὑπολειπτικαὶ εἰσίν, ἀεὶ ἐπὶ τὰ ἑπόμενα τρέχοντες τὰ προηγούμενα δὲ ὑπολείπονται. ἔστι[17] δὲ [[ὁ]] προηγούμενον[18] μὲν οἷον τυχὸν ὁ Κριὸς τὸ ἑπόμενον δὲ οἷον ὁ Ταῦρος, καὶ πάλιν προηγούμενον μὲν ὁ Ταῦρος ἑπόμενον δὲ ὁ Δίδυμος, καὶ ἀεὶ ὁ ἥλιος καὶ ἡ σελήνη ἀπὸ τῶν προηγουμένων ἐπὶ τὰ ἑπόμενα ποιοῦνται τὴν κίνησιν (οἷον ἀπὸ Κριοῦ εἰς Ταῦρον καὶ ἀπὸ τοῦ Ταύρου εἰς τοὺς Διδύμους), καὶ τὰ μὲν ἑπόμενα καταλαμβάνουσι τὰ προηγούμενα δὲ ὑπολείπουσι. διὸ καὶ ὑπολειπτικοὶ λέγονται ὅ τε ἥλιος καὶ ἡ σελήνη, καὶ ὁ τοιοῦτος δρόμος αὐτῶν ὑπόλειψις ὀνομάζεται. προηγητικοὶ δὲ λέγονται οἱ ἄλλοι ἀστέρες, ὁ Κρόνος, ὁ Ζεὺς, ὁ Ἄρης, ἡ Ἀφροδίτη καὶ ὁ Ἑρμῆς, ὅτι οὐ μόνον πρὸς ἀνατολὰς ποιοῦνται τὴν κίνησιν, τουτέστιν ἐκ τῶν προηγουμένων εἰς τὰ ἑπόμενα, ἀλλ᾽ ἔστιν ὅτε καὶ ἐναντίως ἀπὸ τῶν ἑπομένων ἐπὶ τὰ προηγούμενα, καὶ διὰ τοῦτο προηγητικοὶ λέγονται. καλεῖται δὲ ἡ τοιαύτη προήγησις καὶ ὑποποδισμός. |254v

<**9**> πάλιν δὲ τῶν ὑπολειπτικῶν δύο ἀστέρων, ἡλίου τε καὶ σελήνης, ὁ μὲν ἥλιος τὴν μεταξὺ τοῦ ζῳδιακοῦ γραμμὴν δίεισιν, ἣ δὴ καὶ διὰ τοῦτο ἡλιακὴ καλεῖται· οὐ γάρ ἐστιν ὁ ζῳδιακός, ὡς προείρηται, κύκλος ὑπὸ μιᾶς γραμμῆς περιεχόμενος, ἀλλὰ πλάτος ἀξιόλογον ἔχει, ἀλλὰ τὸν μὲν μεσαίτατον [[πάλιν]] τούτου κύκλον, ὃς καὶ μέγιστός ἐστι καὶ εἰς δύο τέμνει τὴν σφαῖραν, ὁ ἥλιος δίεισιν. ἡ δὲ σελήνη κατὰ τὴν τάξιν τῶν ἄλλων πέντε πλανήτων ποτὲ μὲν τὴν μέσην ταύτην γραμμὴν δίεισι ποτὲ δὲ βορειοτέρα ποτὲ δὲ νοτιωτέρα καθίσταται, καὶ ἦν ἂν καὶ αὕτη προηγητικὴ διὰ τὴν τοιαύτην ἀνωμαλίαν κατὰ τὴν τάξιν τῶν ἄλλων ἀστέρων· ἀλλ᾽ ὅτι τὸ μέγεθος τοῦ κύκλου αὐτῆς ὡς προσγειοτάτης τῶν ἄλλων ἔστιν ἐλάχιστον, φθάνει ὑπολειπτικὴ τίς γινομένη, ἀλλ᾽ οὐ προηγητική. [[οὗτοι μὲν οὖν οἱ δύο κύκλοι καὶ τῇ θέσει καὶ τῷ μεγέθει δέδονται· οὔτε γὰρ οἱ τόποι τῶν κύκλων μεταπίπτο]] οὗτοι οὖν πάντες οἱ τῶν πλανήτων κύκλοι τῇ θέσει μὲν δέδονται – ἀμετάπτωτοι γὰρ οἱ τόποι αὐτῶν – τῷ μεγέθει δὲ οὐ δέδονται· οὐ γὰρ ἔχομεν ἑκάστῳ αὐτῶν ἴσον πορίσασθαι.

[17] ἔστι s.l. corr. εἰσὶ
[18] corr. e προηγούμενα m.1

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<**10**> καὶ ἤδη τοῦτό ἐστι τὸ ἐκ τῆς διαιρέσεως σκέλος $|_{255r}$ τέταρτον. ὁ μὲν γὰρ ἰσημερινὸς καὶ οἱ παρ᾽ ἑκάτερα δύο, ὁ θερινὸς δηλαδὴ τροπικὸς καὶ ὁ χειμερινὸς τροπικός, τῇ τε θέσει καὶ τῷ μεγέθει δέδονται, ὁ ἀρκτικὸς δὲ καὶ ὁ ἀνταρκτικὸς οὔτε τῇ θέσει οὔτε τῷ μεγέθει δέδονται, ὁ ὁρίζων δὲ καὶ ὁ μεσημβρινὸς τῷ μεγέθει μὲν δέδονται τῇ θέσει δὲ οὐ δέδονται, ἐκ τοῦ ἐναντίου δὲ οἱ κατὰ βάθος τῇ θέσει μὲν δέδονται τῷ μεγέθει δὲ οὐ δέδονται. καὶ οὕτω μὲν ἔχομεν τὸ πλῆρες τῆς διαιρέσεως· ὁ δὲ ζῳδιακός, ὥσπερ εἴρηται, τῷ μεγέθει μὲν καὶ αὐτὸς δέδοται τῇ θέσει δὲ καθ᾽ ἑαυτὸν μὲν δέδοται ὡς πρὸς ἡμᾶς δὲ οὐ δέδοται.

<**11**> οἱ δύο δὲ λεγόμενοι κόλουροι εἰσὶ μὲν αὐτὸς ὁ μεσημβρινός – πῶς δὲ ὁ εἷς δύο γίνεται, καὶ πῶς ποτὲ μὲν μεσημβρινὸς λέγεται ποτὲ δὲ κόλουρος ὀνομάζεται, μᾶλλον δὲ κόλουροι, ἤδη ἐροῦμεν. μεσημβρινὸς μὲν γὰρ λέγεται ὅτι ὅταν αὐτόθι ἔλθῃ ὁ ἥλιος, τὸ μέσον τῆς ἡμέρας ἐστι, καὶ γίνεται ὁ ἐπινοούμενος οὗτος κύκλος ὡσανεὶ μεσημβρινός, ὅταν τὸν ἥλιον κατὰ κορυφὴν ἔχωμεν. ἐπεὶ δὲ τοῖς μὲν τὴν ἡμετέραν εὔκρατον οἰκοῦσι τὸ μὲν βορειότερον αὐτοῦ μέρος τεθέαται τὸ δὲ νοτιώτερον διὰ τὴν τοῦ κυρτώματος $|_{255v}$ ἐπιπρόσθησιν ἀθέατόν ἐστι καὶ κατὰ τοῦτο τὸ μέρος οἱονεὶ κεκολόβωται, τοῖς δὲ τὴν ἀντεύκρατον πάλιν οἰκοῦσι τὸ μὲν νοτιώτερον ἐκ τοῦ ἐναντίου τεθέαται τὸ δὲ βορειότερον ἀθέατόν ἐστι διὰ τὴν ἀπὸ τοῦ κυρτώματος τῆς γῆς, ὡς εἴρηται, ἐπιπρόσθησιν, καὶ οἱονεὶ κατὰ τοῦτο καὶ οὗτος κεκολόβωται, διὰ τοῦτο τῷ μὲν ὑποκειμένῳ εἷς ἐστιν ὁ τοιοῦτος μεσημβρινός, τῇ σχέσει δὲ δύο, καθάπερ καὶ ἐπὶ τῆς κλίμακος ἔχει· καὶ γὰρ αὕτη τῷ ὑποκειμένῳ μία καὶ ἡ αὐτή, δύο δὲ σχέσεις ἐν αὐτῇ θεωροῦνται, ἥ τε ἀνάβασις καὶ ἡ κατάβασις. σχέσεις δὲ ὀνομάζονται ὅτι πρὸς ἕτερον λέγονται· ἥ τε γὰρ ἀνάβασις πρὸς τὴν κατάβασιν καὶ ἡ κατάβασις πρὸς τὴν ἀνάβασιν. οὕτω κἀν τῷ μεσημβρινῷ οἱ μὲν οἰκοῦντες τὴν εὔκρατον κόλουρον αὐτὸν ἔχουσιν ὡς πρὸς νότον, οἱ δὲ τὴν ἀντεύκρατον οἰκοῦντες κόλουρον αὐτὸν ἔχουσιν ὡς πρὸς βόρειον μέρος, καὶ οὕτως ὁ εἷς δύο τε γίνεται καὶ ἑτεροιοῦται κατὰ τὸ ὄνομα. κόλουρος δὲ λέγεται ὁ κεκολοβώμενος κατὰ τὸ ἄκρον, ὃ καὶ κόλον ὀνομάζεται, $|_{258r}$ εἴτε ἐπὶ τριγώνων τοῦτο λάβοις εἴτε ἐπὶ πυραμίδων, καὶ πάλιν εἴτε ἐπὶ κύκλων εἴτε ἐπὶ σφαιρῶν· ὁ γὰρ ἐλλιπὴς κύκλος κόλουρος λέγεται, ὥσπερ καὶ τὸ ἐλλιπὲς τρίγωνον. καὶ οὗτοι δὲ οἱ δύο κόλουροι οὔτε τῇ θέσει οὔτε τῷ μεγέθει δέδονται· τῇ μὲν θέσει ὅτι μηδ᾽ ὁ μεσημβρινός – ὁ αὐτὸς γάρ ἐστι μεσημβρινός τε καὶ κόλουρος –· οὐ γὰρ ἀεὶ τὸν αὐτὸν ἔχει τόπον· τῷ δὲ μεγέθει μεσημβρινὸς μὲν ὢν δέδοται· ἔστι γὰρ πλήρης καὶ πολλοὺς ἔχει τοὺς ἴσους· κόλουρος δὲ γενόμενος τῷ μεγέθει οὐ δέδοται· κατὰ γὰρ τὰ διάφορα τῆς γῆς κλίματα διάφορον ἔχει καὶ τὴν κολόβωσιν, δηλονότι ποτὲ μὲν πλείονα ποτὲ δὲ ἐλάττονα.

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Translation of the Text

By the consul of the philosophers

<**1**> Both[19] a point and a line that always hold the same place are said to be given; magnitudes for which we can procure equals are said to be given. What is the meaning of each of these, it will have to be investigated next; one must just know that the geometer expounds the "givens" by abstraction, both most generally and in themselves.[20] As for astronomy, these do not fall outside the range of its subject-matter;[21] for this reason, the consideration of such "givens" is, in this case, quite straightforward too. For we say that the celestial sphere[22] is divided in latitude into five parallel circles—two are those which separate both the always-visible and the invisible <stars>, two others are both the summer tropic and the winter tropic, and the

[19] The first clause of the text also contains the first τε … καί correlative, which I translate "both … and". Three dozen will follow. In this instance, the correlative is an original feature of the definition as we read it in the *Data*; later occurrences in the Text may come from (unconscious) imitation of the Source (see below).

[20] I read ἁπλούστερον as preparing for the next sentence, in which the physical world enters via astronomy. The *Data* contains theorems about generic "magnitudes" (μέγεθη) and about geometric objects such as points, lines, angles, circles, triangles, and quadrilaterals.

[21] Fully-fledged "given"-theorems are proved by Ptolemy in Books I and III of the *Almagest*: see n. 10 above.

[22] The "celestial sphere" will be simply called "the sphere" henceforth. Expositions of the circles on the sphere in standard textbooks of elementary astronomy can be read in Cleomedes, I.1.193–208 Todd [this short account triggered a long scholium by John Pediasimus, see sch. 19 in P. Caballero Sánchez, *El* Comentario *de Juan Pediásimo* a los «Cuerpos celestes» de Cleomedes (Nueva Roma 48), Madrid 2018, 206–214]; Geminus, V; Theon of Smyrna, 129.10–133.25 Hiller; Achilles, sects. 22, 23, 25, 27. The involved editions are R. Todd, *Cleomedis Caelestia (ΜΕΤΕΩΡΑ)*, Leipzig 1990; K. Manitius, *Gemini Elementa Astronomiae*, Lipsiae 1898; E. Hiller, *Theonis Smyrnaei philosophi platonici Expositio rerum ad legendum Platonem utilium*, Lipsiae 1878; E. Maass, *Commentariorum in Aratum reliquiae*, Berolini 1898, 25–75. Translations, with a commentary, of Geminus and of Theon of Smyrna can be found in J. Evans, J.L. Berggren, *Geminos's* Introduction to the Phenomena. *A Translation and Study of a Hellenistic Survey of Astronomy*, Princeton – Oxford 2006, and F.M. Petrucci, *Teone di Smirne. Expositio rerum mathematicarum ad legendum Platonem utilium. Introduzione, Traduzione, Commento* (*Studies in Ancient Philosophy*, 11), Sankt Augustin 2012, respectively.

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equator between these—in longitude, by the so-called twelve zodiacal signs—which, once held together and united, complete the zodiacal and oblique circle—in depth,[23] the sphere is divided by the circles of the seven planets.

<**2**> There are also other circles, the horizon and the meridian and the colures, which also appear to divide the celestial whole in latitude.[24] Those of them as bisect the sphere are the greatest among the others and equal to one another; for, as in the case of a circle a straight line drawn through the centre, which we also call "diameter", is the greatest among the straight lines on either side, so in the case of a sphere too the circle that bisects the sphere is the greatest among the circles on either side.[25] These are the horizon and the equator, the meridian and the zodiac; for all of these bisect the sphere and are equal.

<**3**> Now, the equator is given both in position and in magnitude, in position because it always holds the same place; for it was said in the definitions that points and lines that always hold the same place are said to be given,[26] and a circle is also a line,[27] and where is a line, there is also a point, not the inverse, of course.[28] How come the

[23] The "depth" is τὸ βάθος. A "third dimension" is needed because the motions of the planets take place on spheres whose distances from the Earth vary; see Theon of Smyrna, *passim* 170–200 Hiller. Later, the Text will list the planets in the traditional order of decreasing size of the spheres by which they are carried.

[24] The Text apparently wanted to insert any circle on the sphere in one of the categories "latitude", "longitude", and "depth", and in this case he chose the less unsuitable. As no mention of such a typology is found in the extant textbooks of elementary astronomy from Greek antiquity, the Text almost certainly improvised on this point. This confirms its systematic aims.

[25] The theorem for the circle is Euclid, *Elem*. III.7. There is no analogous theorem in Theodosius' *Sphaerica* (the standard textbook on spherics). As is clear from the formulation, the Text is arguing by analogy.

[26] Again, the Text does not realize that a definition of "being given in position" is required.

[27] Strictly speaking—that is, according to *Elem*. I.def.15—a circle is a figure, not a line. This harmless abuse of terminology is current in mathematical and astronomical texts; a striking case is *Elem*. III.

[28] This side remark is reminiscent of the hierarchies of geometric objects debated within ancient philosophical schools. The issue is whether point is prior to line, line to surface, and surface to solid, or vice versa; of course, the whole issue hinges on the meaning of "prior": see the discussion in Proclus' commentary on Book I of the *Elements*, in G. Friedlein, *Procli diadochi in primum Euclidis Elementorum librum commentarii*, Lipsiae 1873, 85–93.5; Ari-

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equator always holds the same place (for a position occurs in a place)? Because this passes through Aries and Libra, and Aries and Libra are unchanging and such as always to hold the same place.[29] For all zodiacal signs are as it were stuck in the sky, and are immovable, so that if the equator is conceived as attached to the immovable constellations, it is also immovable and always holds the same place, and it is given in position. It is also given in magnitude, as said; <u>for it is possible to procure <a circle> equal to it, not only the horizon but also the meridian and the zodiac</u>—actually, the very middle of it: for the other circles are conceived as linear and breadthless, whereas the zodiac is also said to have a sizeable breadth, so that only the very middle circle in it is found to bisect the sphere.[30]

<**4**> <u>Moreover, the <circles> on either side of it, both the summer tropic and the winter tropic, are also given in position and in magnitude</u>, in position because neither of them oversteps the zodiac; for the former is assumed <to be placed> at the beginning of Cancer and as it were in its first minute or point,[31] and with this beginning or first minute—at which the greatest of all days also takes place—the summer tropic happens to take place. If the zodiac, and Cancer in it, and the beginning of Cancer, are such as to be immovable and always to hold the same place, therefore the summer tropic is also given both in place and in its position.[32] In this way, the winter tropic is also given both

stotle's position is discussed in Ch. Pfeiffer, *Aristotle's Theory of Bodies*, Oxford 2018, 85–120. The mathematical counterpart of such debates is the progression of the definitions that open Book I of the *Elements*.

29 If the daily motion of the sphere has to be taken into account—as for instance it has to in the case of the ecliptic—Aries and Libra do not always hold the same place, whereas the equator does. Aries and Libra are the signs whose beginning coincides with the intersection between the equator and the ecliptic.

30 It will eventually become clear that this "very middle" (μεσαίτατος) circle is the yearly circuit of the Sun. On the zodiac being a belt, see Cleomedes, I.2.43–59 Todd (Cleomedes repeatedly uses μεσαίτατος); Geminus, V.51–53; Theon of Smyrna, 133.17–25 Hiller; Achilles, sect. 23.

31 The Sun passes through this point at the summer solstice; the identification of this point as the "first degree" or "first minute" is incorrect but currently used. The beginning of Capricorn marks the winter solstice. The Text's argument exhibits the same drawback as the one showing that the equator is given in position.

32 This hendiadys means "it is given as to place and hence in position". The Text is here blurring the distinction between *definiens* and *definiendum*.

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in place and in position; for both it is conceived and takes place in the first minute of Capricorn. Likewise, these two circles are also given in magnitude; for it is possible to procure both the winter tropic equal to the summer tropic, and vice versa the summer tropic to the winter tropic; for those <circles> are also equal whose distances from the zodiac are equal. Summarizing, these three, the equator and the summer tropic and the winter tropic, are given both in position and in magnitude, exactly as said.

<**5**> On the contrary, the Arctic and Antarctic <circles>, that is, both the one that separates the always-visible and the one <that separates> the invisible <stars>, are given neither in position nor in magnitude. There are even configurations in which neither the invisible <circle> nor the other, always-visible one exist at all; for in the torrid and uninhabited zone the horizon passes through both poles,[33] and all stars, both those farther north and those farther south, both rise and set, so that neither those farther north are always-visible nor those farther south are invisible. Towards the terrestrial latitudes lying on either side, where there are both northernmost and southernmost <circles>,[34] in our temperate zone the always-visible circle is the northernmost one, such as to have the pole just a little elevated and to determine a small always-visible circle, whereas the invisible circle is the southernmost one, such as to have the horizon just a little elevated above the pole and to determine a small circle of invisible <stars>. Since towards our temperate zone the terrestrial latitude is cut by seven parallel latitudinal belts,[35] for every such belt, if we move farther north both the north pole becomes

[33] Strictly speaking, this is true only of locations on the terrestrial equator.

[34] My translation is here tentative; the phrase παρ᾽ οἷς comes directly from the Source, where it refers to people located at a specific latitude; here, as elsewhere in the Text, I have translated παρ᾽ οἷς as a locative. Section 5 describes the way the Arctic and the Antarctic circles change as long as the observer moves from the equator to the poles. These two circles delimit the regions of the celestial sphere whose points, during the daily motion, never set below (resp. rise above) the horizon. As the daily motion takes place around the poles of the equator, to a given observer the points of the celestial sphere that never set (resp. raise) are delimited by a circle parallel to the equator and tangent to the horizon. These two circles, the so-called "Arctic" and "Antarctic" circles, are symmetric with respect to the equator. By their definition, these circles depend on the position of the observer. In particular, they coincide with the horizon (and hence to one another) if the observer is placed at the poles; they do not exist if the observer is placed on the equator. Of course, no observer can ever "see" the Antarctic circle.

[35] The reference to the seven *klimata* is just parade here.

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more elevated and the circle of the always-visible <stars> becomes greater. Likewise, the south pole descends further and the horizon becomes higher, and proportionally the circle that determines the invisible <stars> also becomes greater. Then, in themselves both the Arctic and the Antarctic <circle> are given neither in position nor in magnitude—for they change both their position and their magnitude according to the different locations and the different latitudinal belts of the Earth—whereas with respect to one another they change neither their position nor their magnitude, but are as it were also given as a pair;[36] for whenever the always-visible circle is the northernmost and the smallest, then the invisible circle is also the southernmost and the smallest, and on the contrary, whenever the always-visible <circle> is the greatest, the invisible one is also the greatest and is both similar in position and equal in magnitude, and proportionally in the place between both the smallest and the greatest <circles>, so that in themselves these two circles, the Arctic and the Antarctic, are given neither in position nor in magnitude, whereas with respect to one another are given both in position and in magnitude. And so much about the parallels.

<**6**> The zodiac itself is also given in itself both in position and in magnitude;[37] for both it is always found to hold the same place and it is possible for us to procure another <circle> equal to it, either the equator or the meridian or the horizon; for all these bisect the sphere. It was said "in itself" because it does not always keep the same place with respect to us; for it is an oblique <circle>, not a parallel, and it is partly farther north partly farther south, and with respect to the upright sphere it is partly more slanted partly more upright, that is, partly more acute-angled partly more right-angled partly more obtuse-angled. For, since the equator is a sign that the sphere is upright and the equator is cut by the zodiac, the obliquity is more acute-angled near to the section, whereas the farthest away from the section it is as it were obtuse-angled, and between these it is as it were right-angled.[38] Then, in this way about the zodiac too, which also cuts the sphere in longitude, as said.

[36] I take this to be the most interesting remark of the Text.

[37] The ecliptic is here intended.

[38] This unfitting terminology conveys the idea that the greatest angle between the ecliptic and the equator occurs where they intersect (these are the equinoctial points), whereas the ecliptic is in a sense "parallel" to the equator at the solstitial points.

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<**7**> Since[39] a complete division does not require only the affirmative <statement> made of both <statements>, such as “being given both in position and in magnitude”, nor does it <require> only the negative one made of both <statements>, such as “being given neither in position nor in magnitude”, but also the one mixing the two, such as “being given in position and not in magnitude” or “being not given in position but in magnitude”, let us also see in what heavenly items these can be found. Then, the meridian and the horizon are not given in position but are given in magnitude; for their place changes according to the different locations; for neither the Earth nor its surface are even and flat, but spherical and uneven. For this reason, the horizon is not the same everywhere, but in some places it is lower in some places it is higher, nor is the meridian the same everywhere; for neither the rising and setting are the same everywhere because of the intervening longitudinal convexity[40] of the Earth, nor is the culmination the same: for the culmination of the Sun indicates the meridian, whenever the Sun is above our head. These <circles> are also given in magnitude, for it is possible to procure both the meridian equal to the horizon and the horizon to the meridian. So much about these matters too.

<**8**> Next, let us investigate the circles according to depth. These are seven, the <circle> of Saturn, of Jupiter, of Mars, of the Sun, of Venus, of Mercury and of the Moon. All the said heavenly bodies carry a specific and deliberate[41] motion, yet not all in similar ways. For the Sun and the Moon are behind-leaving, as they always move towards what follows and leave what precedes behind. These are, to take a chance instance, Aries preceding and Taurus following, and again Taurus preceding and Gemini following, and the Sun and the Moon always make their motion from what precedes towards what follows

[39] The discussion in this paragraph is framed in the scheme of the several conjunctions of two predicates and of their negations. The conjunction “being given in magnitude but not in position” is treated here; the next two paragraphs will show that the circles of the planets are given in position but not in magnitude.

[40] I translate the hendiadys κυρτώματα καὶ ἐγκλίματα by “longitudinal convexity”. The idea is that midday is the middle point of the time interval between the Sun’s rising and setting.

[41] The qualifier “deliberate” (προαιρετικός) for the “motion” (κίνησις) of the planets is standard Stoic doctrine, see Cleomedes, I.2.8,24,53, I.4.1, II.1.304, II.6.25 Todd (and consequently John Pediasimus, see sch. 19, lines 62 and 67 in Caballero Sánchez, *El* Comentario [cit. n. 22]); Geminus, XII.24, 144.16 Manitius; Theon of Smyrna, 201.20 Hiller. This paragraph contains a preliminary clarification of the terms that describe the direction of the motion of the heavenly bodies.

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(such as from Aries towards Taurus and from Taurus towards Gemini) and they overtake what follows and leave what precedes behind. For this reason, the Sun and the Moon are also called "behind-leaving",[42] and such a course of them is named "behind-leaving". The other heavenly bodies are called "forward-carrying", namely, Saturn, Jupiter, Mars, Venus and Mercury, insofar as they not only make their motion towards the East, that is, from what precedes towards what follows, but sometimes, inversely, they also <make a motion> from what follows towards what precedes, and for this reason their are called "forward-carrying". Such forward-carrying is also called "retrogradation".

<**9**> Again, of the two behind-leaving heavenly bodies, namely, both the Sun and the Moon, the Sun passes through the line between the zodiacal signs, which for this very reason is also called "heliacal"; for, as said above, the zodiac is not a circle contained by one single line, but has a sizeable breadth, yet the Sun passes through the very middle circle of it, which is also a great <circle> and bisects the sphere. On the contrary, the Moon, in agreement with the characteristic feature of the other five planets, sometimes passes through this middle line, sometimes it lies farther north or farther south, and it may happen that it is also forward-carrying because of such an anomaly, in agreement with the characteristic feature of the other heavenly bodies; nevertheless, as the size of its circle is the smallest one on account of its being nearer to the Earth than the other <planets>, what prevails is its being on the whole behind-leaving, and not forward-carrying.[43] Then, all these circles of the planets are given in position—for their places are immovable—but they are not given in magnitude;[44] for we cannot procure <a circle> equal to each of them.

[42] A motion "from what precedes to what follows" (what precedes and follows are the signs) is an eastward motion because the direction of "precedence" is set by the daily motion. For the qualifiers "behind-leaving" (ὑπολειπτικός) and "forward-carrying" (προηγητικός) and related verbs see Geminus, XII.20–26, 142.20–144.25 Manitius; Theon of Smyrna, 147.8 Hiller (and more generally 147.7–148.12); Ptolemy, *Almagest* I.8 and XII.1, in Heiberg, *Claudii Ptolemaei opera* [cit. n. 7], I.1, 27.17–28.4, and I.2, 456.8–9, 459.5–6, 463.5.

[43] Thus, the anomaly of the Moon is not large enough to make it retrograde. That the Moon does not have a retrograde motion *because* it is the planet nearest to the Earth is false.

[44] This sentence is preceded by a deleted alternative to it, reading "Then, these two circles are given both in position and in magnitude—for the places of the circles do not mov" (the sentence breaks in the middle of a word). Clearly, the deleted sentence is at variance with the categorization the "circles in depth" are assigned in the author's fourfold predicate scheme. I shall comment on the implications of this erasure at the very end of the paper.

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<**10**> And this is already the fourth leg of the division.[45] For the equator and the two <circles> on either side, clearly the summer tropic and the winter tropic, are given both in position and in magnitude, the Arctic and the Antarctic <circle> are given neither in position nor in magnitude, the horizon and the meridian are given in magnitude but are not given in position, and vice versa, the <circles> in depth are given in position but are not given in magnitude. And in this way we have got a complete division; the zodiac, exactly as said, is itself also given in magnitude but, as for position, it is given in itself but it is not given with respect to us.

<**11**> The two so-called colures are indeed the meridian itself—how come the one becomes two, and how come sometimes it is called "meridian" sometimes it is named "colure", better still, "colures", we shall say now. For it is called "meridian" because, exactly as the Sun comes <on it>, it is midday, and this imaginary circle becomes as it were a "mid-day" one, whenever we have the sun on the vertical. Since, for those who inhabit our temperate zone, the northern part of it turns out to be visible whereas the southern one is, because of the interposition of the convexity of the Earth, invisible and as it were curtailed of this part, while, again, for those who inhabit the opposite of our temperate zone, vice versa the southern <part> turns out to be visible whereas the northern one is invisible because, as said, of the interposition of the convexity of the Earth, and this is as it were curtailed of this part too, for this reason such a meridian is one as for its matter, but relationally it is two, very much as it is for a ladder too; for this is also one and the same as for its subject, but two relations can be observed in it, namely, both ascent and descent.[46] They are named "relations" because they are

45 The fourfold predicate scheme is now complete; this paragraph summarizes the findings. The final paragraph will expound a different sense in which a circle can be not given in magnitude, namely, by truncation. Note the brief digression on the meaning of "relation" and the terminological point about colures, the other objects in the simile being taken from geometry.

46 The simile of the ladder adapts the standard example of "heteronyms" in the Aristotelian commentators: see Simplicius *in Cat. 1* (*CAG*, 8), 22.30–33 Kalbfleisch, and Philoponus *in Cat. 1* (*CAG*, XIII.1), 14.17–22 and 15.4–10 Busse (ascent and descent are names that make the σχέσις actualized by a ladder clear), whose source is Ammonius *in Cat. 1* (*CAG*, IV.4), 16.24–29 Busse; and Alexander *in Top. V.4* (*CAG*, II.2), 398.2–5 Wallies. The same example is found in John Damascenus' compendium *Dialectica*, 14.18–20 and 18.2–4, in B. Kotter, *Die Schriften des Johannes von Damaskos* (Patristische Texte und Studien 7), I, Berlin 1969.

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said with respect to another item: for both ascent <is said> with respect to descent and descent with respect to ascent. In this way, in the case of the meridian too, those who inhabit the temperate zone have it truncated towards south, whereas those who inhabit the opposite of the temperate zone have it truncated towards the northern part, and in this way the one both becomes two and becomes other as of its name.[47] A <circle> curtailed of its extreme is called "colure",[48] which is also named "truncated", whether you apply it to triangles or to pyramids, and again whether to circles or to spheres; for a truncated circle is called "colure", exactly as a truncated triangle is too. And these two colures are given neither in position nor in magnitude: <it is not given> in position because the meridian is not either—for the meridian is the same as a colure—for it does not always hold the same place; <a colure> is given in magnitude to the extent that it is the meridian, for it is entire and it does have many <circles> equal <to it>; however, when it becomes a colure it is not given in magnitude; for depending on the terrestrial latitude it also has a different curtailing, sometimes more sometimes less,[49] as is clear.

The Source of the Text

The Source of the Text is unquestionably Theon of Smyrna, *Expositio*, 132.5–133.16 Hiller.[50] Let us read this passage, followed by a translation:[51]

ἀλλ᾽ ὁ μὲν ἰσημερινὸς καὶ οἱ ἑκατέρωθεν τούτου τροπικοὶ δεδομένοι καὶ ἀραρότες τοῖς μεγέθεσι καὶ ταῖς θέσεσι. δεδόσθαι δὲ λέγεται τῇ θέσει σημεῖά τε καὶ γραμμαὶ ἃ τὸν αὐτὸν ἀεὶ τόπον ἐπέχει· τῷ δὲ μεγέθει δεδομένα χωρία τε καὶ γραμμαὶ καὶ γωνίαι λέγονται οἷς δυνάμεθα ἴσα πορίσασθαι {→ 1}. ὁ δὲ τοῦ ἰσημερινοῦ κύκλος καὶ

47 Compare this argument with sch. 19, lines 96–103 in Caballero Sánchez, *El* Comentario [cit. n. 22].

48 For "colure" as the denomination of a genus see e.g. Geminus, V.49; Ptolemy, *Almagest* II.6, in Heiberg, *Claudii Ptolemaei opera* [cit. n. 7], I.1, 103.5.

49 This is false, for both colures and the horizon are great circles.

50 Theon certainly found this passage in Adrastus, his source here. Why Adrastus compiled it, and from what source, will remain a mystery. For it is obvious that this passage disrupts Theon's argument: read the bewilderment in Petrucci, *Teone di Smirne* [cit. n. 22], 454–456.

51 References are made to the sections of the Text where the marked arguments are discussed; the sign "/" means that the associated argument is not used in the Text. Two of the atheteses and the integration are Hiller's.

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οἱ ἑκατέρωθεν τροπικοὶ ἀεὶ τὸν αὐτὸν ἐπέχουσι τόπον καὶ ἀραρότες εἰσί, καὶ ἴσους αὐτοῖς οἷόν τε πορίσασθαι, τῷ μὲν ἰσημερινῷ τόν τε ζῳδιακὸν καὶ τὸν ὁρίζοντα καὶ τὸν μεσημβρινόν, τῷ δὲ χειμερινῷ τὸν θερινὸν καὶ τῷ θερινῷ τὸν χειμερινόν, οἵτινες διὰ τούτων ἀεί εἰσι δεδομένοι {→ 3, 4}, ὅτι οὐκ ἐφ᾿ ἡμῖν ἐστι τοιούσδε ἢ τηλικούσδε ὑποστήσασθαι αὐτούς, ἀλλὰ τῇ φύσει ὑποκείμενοι τοιοῦτοι καὶ δεδομένοι, κἂν μὴ ἡμεῖς δῶμεν· ἃ δὲ ἐφ᾿ ἡμῖν ἐστι δοῦναι αὐτὰ ἢ τοῖα ἢ τοῖα εἶναι, ταῦτα τῇ {δὲ} φύσει οὐκ ἔστι δεδομένα {→ /}. φύσει οὖν δεδομένοι καὶ ἀραρότες {τουτέστιν ὑφεστῶτες καὶ ἀραρότες} ὅ τ᾿ ἰσημερινὸς καὶ οἱ ἑκατέρωθεν καὶ τῇ θέσει καὶ τοῖς μεγέθεσιν {→ 3, 4}. ὁ δὲ ζῳδιακὸς τῷ μὲν μεγέθει δέδοται καὶ τῇ κατ᾿ αὐτὸν τὸν οὐρανὸν θέσει, τῇ[52] δὲ πρὸς ἡμᾶς οὐ δέδοται {τῇ} θέσει· μεταπίπτει γὰρ ὡς πρὸς ἡμᾶς διὰ τὴν ἐν τῷ παντὶ λόξωσιν ἄλλοτε ἄλλως ἱστάμενος ὑπὲρ ἡμᾶς {→ 6}. μεσημβρινὸς δὲ καὶ ὁρίζων τῷ μὲν μεγέθει δεδομένοι – μέγιστοι γάρ – τῇ δὲ θέσει μεταπίπτοντες καθ᾿ ἕκαστον κλίμα τῆς γῆς ἄλλοι παρ᾿ ἄλλοις γινόμενοι· οὔτε γὰρ ἅπασι τοῖς ἐπὶ τῆς γῆς ὁ αὐτὸς ὁρίζων, οὔτε πᾶσι τὸ αὐτὸ μεσουράνισμα, οὔθ᾿ ἑκάστῳ ἐστὶν ὁ <αὐτὸς> μεσημβρινός {→ 7}. οἱ μέντοι πρὸς τοῖς πόλοις, ὅ τε ἀρκτικὸς καὶ ὁ ἀνταρκτικός, οὔτε τοῖς μεγέθεσι δέδονται οὔτε ταῖς θέσεσι· κατὰ δὲ τὴν διαφορὰν τῶν νοτιωτέρων καὶ βορειοτέρων κλιμάτων παρ᾿ οἷς μὲν μείζονες παρ᾿ οἷς δὲ ἐλάττονες ὁρῶνται, καὶ κατὰ μέσην μέντοι τὴν γῆν, τουτέστι κατὰ τὴν ὑπὸ τὸν ἰσημερινὸν λεγομένην ζώνην διὰ καῦμα ἀοίκητον, οὐδ᾿ ὅλως γίνονται, τῶν πόλων ἀμφοτέρων ἐκεῖ φαινομένων καὶ τοῦ ὁρίζοντος δι᾿ αὐτῶν ἐκπίπτοντος {→ 5}. εἰσὶ δὲ οἳ καὶ τὴν σφαῖραν ὀρθὴν καλοῦσι, πάντων τῶν παραλλήλων ὀρθῶν γινομένων ὡς πρὸς ἐκείνους τοὺς τόπους τῆς γῆς {→ /}.

Now, the equator and the tropics on either side of this are given and fixed in magnitudes and in positions. Both points and lines that always hold the same place are said to be given in position; both regions and lines and angles for which we can procure equals are said to be given in magnitude {→ 1}. The circle of the equator and the tropics on either side always hold the same place and are fixed, and it is possible to procure <circles> equal to them, both the zodiac and the horizon and the meridian to the equator, the summer <tropic> to the winter <tropic> and the winter one to the summer one, which for these reasons are always given {→ 3, 4}, insofar as it is not up to us to establish them of such-and-such a kind or of such-and-such a size, being on the contrary laid down and given such-and-such by nature, even if we do not make them; what is up to us to make be as they are or such or such, this is not given by nature {→ /}.

52 τῷ cod.

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Then, both the equator and the <tropics> on either side are given and fixed by nature both in position and in magnitudes {→ 3, 4}. The zodiac is given in magnitude, and in position with respect to the heaven itself, whereas it is not given in position with respect to us; for, because of its obliquity in the whole, it continuously changes from here to there with respect to us while staying over us {→ 6}. The meridian and the horizon are given in magnitude—for they are great <circles>—while moving in position for each terrestrial latitude and continually becoming other; for the horizon is not the same everywhere on the Earth, nor is the culmination the same everywhere, nor is the meridian the same for each <place> {→ 7}. As for the <circles> near to the poles, both the Arctic and the Antarctic, they are given neither in magnitudes nor in positions; according to the difference of farther north or farther south terrestrial latitudes, they are seen greater by some and lesser by others, and in the middle of the Earth, that is, in the so-called zone under the equator, uninhabited because of heat, they do not exist at all, both poles being there visible and the horizon falling through them {→ 5}. There are also people who call it "upright sphere", all parallels becoming upright in those places of the Earth {→ /}.

The Relation of the Text with the Source

The Text is six and a half times longer than the Source, which contains only statements about specific circles being given and succinct reasons for this being so. As the Source gives the overall line of the argument, the fourfold predicate scheme included, the Text modifies it by selection, inflation, and complement. Selection means that something is omitted. Inflation means that one-sentence explanations are expanded. Completion means that further material is added.

The omissions are easy to detect: these are the terminological point about the "upright sphere"—a denomination that occurs in the Text though (lines 69–70 and 72)—and, most importantly, the explanation of the meaning of "by nature". Accordingly, the Source's intimation that something is given "by nature" is eliminated. Likewise, the incongruous participle "fixed" (ἀραρότες), almost replacing "given" when position is at issue, is eliminated. The Text also corrects the several incongruous plurals τοῖς μεγέθεσι and ταῖς θέσεσι found in the Source. Contrary to the Text, the Source gives exactly the Euclidean definitions of "given in position" and "given in magnitude", even if in inverse order, a feature that will be kept in the Text.

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Inflated material is found in sects. 1 and 3–7, where the circles on the celestial sphere are discussed. In these sections, all proofs of why a specific circle is given in a specific sense can be taken to elaborate on the Source's proofs.

As for the added material, this amounts to the following items: sects. 8–11, featuring the motions of the planets, a recapitulation that puts emphasis on the fourfold predicate scheme, and the colures; the introductory remarks; a number of short digression (great circles are those that bisect the sphere, explanations of the logic of the fourfold predicate scheme) and of terminological clarifications; and the material Theon of Smyrna (for instance, the definitions of the several circles on the sphere) provides elsewhere.

Lexical modifications include ἑκατέρωθεν → παρ' ἑκάτερα and the forms ἔχει / κατέχει instead of the canonical ἐπέχει in the definition of "given in position". Finally, the massive presence of τε … καί and of παρ' οἷς … παρ' οἷς correlatives in the Text (lines 2, 8, 12, 17, 31, 36–38, 39, 41–43, 46, 48, 53, 55, 57, 61, 62, 64–66, 87, 97, 103, 117, 132, 133, 136, 141; and 48, 69–71, 83, respectively) appears to be imitative of the Source (lines 2, 3, 5, 17; and 18, respectively).

If the Text was an introductory lecture, and despite its length, it appears to be better organized than the Source, whose presence in Theon's exposition disrupts the main line of discourse.

The Author of the Text

The identity of the author of the Text can at best be the object of an informed guess. What we know is that the author held the charge of "consul of the philosophers" [53] and that he was able to appropriate a non-trivial mathematical argument.

The most "natural" candidate as the author of the Text is John Pediasimus, who held the charge of "consul of the philosophers" at the end of the 13th century and who is well known for his mathematical penchant.[54] In particular, Pediasimus contributed several

[53] On the "consul of the philosophers", a charge activated in Michael Psellus' times, see C.N. Constantinides, *Higher Education in Byzantium in the Thirteenth and Early Fourteenth Centuries (1204 –* ca.*1310)*, Nicosia 1982, 113–132.

[54] For Pediasimus, see Constantinides, *Higher Education* [cit. n. 53], 116–125; D. Bianconi, *Tessalonica nell'età dei Paleologi. Le pratiche intellettuali nel riflesso della cultura scritta* (Dossiers Byzantins 5), Paris 2005, 60–72; I. Pérez Martín, "L'écriture de l'*hypatos* Jean Pothos

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scholia to Cleomedes' *Cyclic Theory*,[55] even if nothing in them suggests that Pediasimus might have written the Text. However, this supposition is somehow corroborated by the fact that the Text has been copied without mistakes, something which is more likely to happen if the copy closely follows the first redaction. Despite a couple of textual affinities that might be more than coincidences,[56] my resistance to endorsing Pediasimus' name comes from the fact, repeatedly confirmed by a careful reading of his scientific production, that he was a dull compilator who frequently mars his arguments with gross misunderstandings.[57] As we have seen, the Text deals with a tricky matter and does not contain any. To put it more charitably, I do not recognize Pediasimus' style in the Text.[58]

A less obvious candidate is John Italos,[59] who was appointed "consul of the philosophers" after Michael Psellus. Italos wrote, in addition to the *Quaestiones quodlibetales*—a collection of *adversaria* possibly connected to his teaching—short texts on logic, dialectic, and rhetoric, and a commentary on Book II–IV of Aristotle's *Topics*, where he plunders Alexander of Aphrodisias. As the Text puts more emphasis on the combinatorics of complex predicates than the Source does, the focus on logic a part of Italos' production carries somehow corroborates the hypothesis that he was the author

Pédiasimos d'après ses scholies aux *Elementa* d'Euclide", *Scriptorium* 64 (2010), 109–119; F. Acerbi, I. Pérez Martín, "Les études géométriques et astronomiques à Thessalonique d'après le témoignage des manuscrits: de Jean Pédiasimos à Démétrios Kydônès", *Byzantion* 89 (2019), 1–35: 3–7. See also *PLP*, nr. 22235.

55 These scholia are edited in Caballero Sánchez, *El* Comentario [cit. n. 22].

56 See n. 41 and 47 above, in particular the latter.

57 One is reminded of A.J.H. Vincent's hesitation about publishing Pediasimus' *Specific Remarks* on music: "Nulle part le précepte Συναγάγετε τὰ περισσεύσαντα n'a besoin d'être invoqué plus qu'ici, pour motiver, en quelque sorte, la publication d'un traité où l'on trouve des idées aussi fausses et des erreurs aussi grossières" [A.J.H. Vincent, *Notice sur divers manuscrits grecs relatifs à la musique, comprenant une traduction française et des commentaires* (Notices et extraits des manuscrits de la Bibliothèque du Roi et autres bibliothèques 16.2), Paris 1847, 289].

58 Compare the Text with sch. 19 in Caballero Sánchez, *El* Comentario [cit. n. 22].

59 On Italos, see A. Rigo, "Giovanni Italo", *Dizionario Biografico degli Italiani* 56 (2001), available online at https://www.treccani.it/enciclopedia/giovanni-italo_(Dizionario-Biografico)/. For Italos' commentary on the *Topics*, see S. Kotzabassi, *Byzantinische Kommentatoren der aristotelischen Topik. Johannes Italos & Leon Magentinos*, Θεσσαλονίκη 1999, in particular 27–38.

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of the Text.[60] A further, yet partial, corroboration comes from the circumstance that Italos did read Theon of Smyrna, as the *Quaestio quodlibetalis* nr. 82 shows.[61] This is only a partial corroboration because Italos appropriated a text contained in the first part of the *Expositio*, where arithmetic and harmonic theory are expounded. This part was not handed down by the line of tradition that transmitted the second part, entirely devoted to astronomy: while the first part is witnessed as early as the manuscript Venezia, Biblioteca Nazionale Marciana, gr. Z. 307 (coll. 1027; 12th c.; *Diktyon* 69778), the second part surfaces only in Marc. gr. Z. 303 (coll. 534; here mid 14th c.; *Diktyon* 69774). As a portion of the first part is also handed down by an independent line of transmission, it is not unlikely that the traditions of the two main parts of Theon's *Expositio* have split in late antiquity. If this has been the case, and as Italos lived before the above-mentioned manuscripts were copied, his acquaintance with the first part does not necessarily entail that he also read the second part.

A third, much more shadowy, candidate is Kyprianos (*ca*. 1300; *PLP*, nr. 13944), an addressee of Nikephoros Choumnos. If we accept the identification of the consul of the philosophers Kyprianos with the *chartophylax* of the Great Church Niketas Kyprianos, his interest in astronomy is testified by its owning the manuscript Firenze, Biblioteca Medicea Laurenziana, Plut. 28.39 (end 9th – beginning 10th c.; *Diktyon* 16220), which is the earliest surviving witness of Hipparchus' commentary on the *Phaenomena* of Aratus and Eudoxus.[62]

There is a fact that both complicates and simplifies matters, for it suggests that the three candidates introduced so far are unlikely to have written the Text. Towards the end of sect. 9 of the Text (lines 111–112), the copyist deletes a sentence (which so formulated is at variance with the ongoing argument) and rewrites it. Under normal conditions, an intervention of this kind should be regarded as an authorial correction. Could the nameless copyist be the nameless "consul of the philosophers" himself? If this is the case, and if we suppose to play a complete information game, John Ampar [Emparis] (*PLP*, nr. 800), who held the charge in 1351–54, might be a good bet.

60 However, neither in Italos' *Quaestiones quodlibetales* nor in his opuscules can any text be found that even resembles the Text. I have checked the editions G. Cereteli, *Iohannis Itali opuscula selecta*, I–II, Tbilisi 1924–26; P. Joannou, *Ioannes Italos*, *Quaestiones quodlibetales*, Ettal 1956.

61 See D. O'Meara, "Empédocle Fragment 143: Un nouveau témoignage chez Jean Italos", *Revue des Études Grecques* 123 (2010), 877–879.

62 The ownership is recorded on f. 130v of the manuscript. Hipparchus' text is edited in K. Manitius, *Hipparchi in Arati et Eudoxi Phaenomena commentariorum libri tres*, Lipsiae 1894.

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