

***THE TEXTUAL TRADITION OF NICOMACHUS' INTRODUCTIO
ARITHMETICA AND OF THE COMMENTARIES THEREON:
A THEMATIC CROSS-SECTION***

FABIO ACERBI

CNRS, UMR8167 Orient et Méditerranée, équipe "Monde Byzantin"
52 Rue du Cardinal Lemoine, Paris (France)
fabacerbi@gmail.com

Abstract

The entire textual tradition of Nicomachus' *Introductio arithmetica* and of the commentaries and scholia thereon is investigated by using a single, unusually cryptic sentence in Nicomachus' text as case-study. The relations between the "Recensions" of Ammonius' lecture notes are also clearly established. Finally, the way an entire exegetical tradition has dealt with such a cryptic sentence is discussed.

Keywords: Nicomachus, Greek arithmetics, manuscript tradition, commentaries

Resumen

El presente artículo ofrece un estudio completo de la tradición textual de la *Introductio arithmetica* de Nicómaco, incluidos los comentarios y escolios a la obra, a partir del estudio de caso de una sola frase, inusualmente críptica, del texto de Nicómaco. Asimismo, se delimitan con claridad las relaciones entre las diferentes "recensiones" de las notas de lectura de Amonio. Para finalizar, se analiza la forma en que la tradición exegetica ha lidiado con esta críptica sentencia.

Metadada: Nicómaco, aritmética griega, tradición manuscrita, comentarios

THE TEXTUAL TRADITION OF NICOMACHUS' INTRODUCTIO ARITHMETICA AND OF THE COMMENTARIES THEREON: A THEMATIC CROSS-SECTION

FABIO ACERBI*

1. Introduction

Nicomachus' *Introductio arithmetica* (henceforth *Ar.*) and its commentators are a *mare magnum* no one has seriously studied in a complete and thorough way. As we shall briefly see in Section 4, major philological problems –without doubt the most intricate philological problems that affect the entire Greek technical corpus– make such an undertaking extremely difficult. I shall be no exception; I shall take instead an unprecedented route: I will present a thematic cross-section of the entire manuscript and exegetical record by using a single sentence in Nicomachus' text as case-study. Of course, this sentence cannot be a chance one. I have selected an unusually cryptic statement if gauged to Nicomachus' writing standards –so unusually cryptic and unsyntactical as to quite certainly be a gloss that has crept into the text at a deep pre-traditional stage. This happened in fact before the renowned Neoplatonic philosopher Ammonius (early 6th century) held a course on *Ar.* whose lecture notes make, in the form of a number of "Recensions", the bulk of the exegetical record on Nicomachus' treatise. An interesting fact is that a paraphrasis of the test-sentence crept in its turn, as an adjunct (henceforth "Adjunct") to the very last sentence of Book I, into the text transmitted by a fair majority of the available manuscripts.

Thus, the present study has two philological aims and a related historical aim:

- to investigate in depth but locally the entire manuscript tradition of *Ar.*;
- to state clearly what are the relations between the "Recensions" of Ammonius' lecture notes (henceforth "Rec.");

* Reproductions of, and information on, most of the manuscripts I shall mention can be found by searching the standard database <https://pinakes.irht.cnrs.fr> by means of the *Diktyon* number I have indicated either in the list of Nicomachean manuscripts set out in the Appendix or, for those manuscripts that are not included in the list, in the main text and in the footnotes. I thank Sofia di Mambro, Paolo Fait, Luca Farina, Diether Roderich Reinsch, Stefano Serventi, Claudia Sojer, and Francesco Valerio for the logistic support, Ciro Giacomelli, Anna Gioffreda, Inmaculada Pérez Martín, and David Speranzi for the paleographic expertises, Bernard Vitrac for a critical reading of a draft of this paper.

- to present a specimen of the contortions an exegetical tradition undergoes as a reaction to a cryptic text.

I shall set out the philological and exegetical record in Section 5. Before doing this, Section 2 presents the (non-trivial) mathematical background, Section 3 my test-sentence, Section 4 the philological context. On the basis of the documents set out in Section 5, Section 6 will offer a series of philological remarks; in particular, I shall outline a broad picture of the textual tradition of *Ar.* in form of clusters of related manuscripts. It is also to be noted that, if the test-sentence I have selected really is an interpolation, and since it is witnessed to by all manuscripts, the tradition of *Ar.* is rooted in an archetype. I shall also show that the so-called Rec. II of Ammonius' lecture notes is a Byzantine recension of Rec. I, and that Rec. IV is an independently conceived Byzantine commentary.¹ Section 7 will summarize the main points of the interpretive proposals of our test-sentence set forth in the scholarly tradition of commentaries and scholia on *Ar.* Finally, Section 8 will show to what extent the three aims listed above have been attained. The Appendix presents my database, including the list of the manuscripts I could not check.²

The documentary record is huge: I have collated about 100 manuscripts. Since this is not a critical edition, all variant readings must be made available to the reader, even those readings contained in manuscripts that, already at this stage of the research, are patently doomed to be discarded as apographs of extant models. For this reason, I have decided to present the variant readings in a variety of ways, none of which resorts to a canonical critical apparatus; in particular, I make systematic use of paratextual elements such as italics or underlining, and, most importantly, of colour markings. This entails that the present paper makes better sense if published online. Readers may well be disconcerted by my layout choices; I urge them to consider what a traditional critical apparatus would have looked like.

I decided to keep myself “blind” to any feature of the manuscripts I have seen that figures in portions of *Ar.* other than the very end of Book I, where the test-sentence lies.³ Since my aim was not to prepare a critical edition of *Ar.*, I had to set a range of relevant information I allowed myself availing of –I set the strictest one.

¹ For the meaning of “Byzantine recension”, and for a survey of them, see F. Acerbi, “Byzantine Recensions of Greek Mathematical and Astronomical Texts: A Survey”, *Estudios bizantinos* 4 (2016), 133-213.

² The overall numbers are: 83 witnesses of *Ar.* that include the test-sentence; 20 that do not or that I could not check (9 items in the latter category); 19 witnesses of Rec. IV, 2 of which also contain *Ar.*

³ I will thus disregard the results in F. Acerbi, “Eliminazioni diagrammatiche”, *Scripta* 13 (2020), 9-37, in which a wide-ranging campaign of eliminations is carried out by means of special methods. It goes without saying that the two approaches corroborate each other.

2. Mathematical Background

A fragment from Archytas is the first document that attests for investigations into numerical means:⁴

There are three means in music: one is the arithmetic, the second geometric and the third subcontrary, which they call “harmonic”. The mean is arithmetic, whenever three terms are in proportion by exceeding one another in the following way: by that which the first exceeds the second, by this the second exceeds the third. And in this proportion, it turns out that the interval of the greater terms is smaller and that of the smaller greater. The mean is geometric, whenever they [*scil.* the terms] are such that as the first is to the second so the second is to the third. Of these <terms> the greater and the lesser make an equal interval. The mean is subcontrary, which we call harmonic, whenever they are such that, by which part of itself the first term exceeds the second, by this part of the third the middle exceeds the third. It turns out that, in this proportion, the interval of the greater terms is greater and that of the lesser is less.

Thanks to a clever definition not coinciding with Archytas,⁵ mathematicians posterior to him carried out a complete classification of the means, listing eleven of them. The process of classification took a long while; the details of its development vary from source to source: these are Nicomachus, *Ar.* II.21-29; Iamblichus, *in Ar.* III.36-71 Vinel;⁶ Theon of Smyrna, *Expositio*, 106-111 and 113-119 Hiller;⁷ Pappus, *Coll.* III.44-57, who mentions “Nicomachus the Pythagorean”. To all these sources as well as to Archytas, a mean is a triad of numbers or “terms” (c, b, a) , with $a > b > c$ and such that b is univocally determined by a and c ; by metonymy, the middle term b usually receives the same denomination of “mean”.⁸

⁴ Porphyry, *in Harm.* I.5, 93.5-17 Düring = fr. 47 B 2 DK; English translation by C. A. Huffman, *Archytas of Tarentum: Pythagorean, Philosopher and Mathematician King*, Cambridge 2005. The “interval” between the terms is their ratio; exactly this terminological point is discussed by Porphyry in the excursus in which he quotes Archytas and other authorities. This denomination remained standard in harmonic theory.

⁵ Since Archytas’ harmonic mean only refers to the notion of “part” (= a divisor), it does not coincide with the item of the classification that bears the same name (this fact escaped Pappus, who identifies the two means in *Coll.* III.30): they do coincide only if the available ratios are epimoric.

⁶ *Jamblique, In Nichomachi Arithmetica*, ed. N. Vinel, Pisa – Roma 2014.

⁷ *Theonis Smyrnaei Expositio rerum ad legendum Platonem utilium*, ed. E. Hiller, Lipsiae 1878.

⁸ Greek technical jargon calls the triad μεσότης and the middle term μέσος. The ultimate source of all modern accounts of the classification is P. Tannery, “L’arithmétique des grecs dans Pappus”, *Mémoires de la Société des sciences physiques et naturelles de Bordeaux* 3 (1880), 351-371, reprinted in Id., *Mémoires Scientifiques*, I, Toulouse – Paris 1912, 80-105: 90-98.

The means are classified as follows. Take the three differences of the terms, $a - b$, $b - c$, $a - c$, which authors qualified by Pappus as “recent” (*Coll.* III.46) called “first”, “second”, and “third excess”. A mean is determined by identifying the ratio of any two of the three excesses and the ratio of two of the three terms, possibly taking the same term twice. Let us see how only 11 combinations result. Since a ratio can always be taken to be greater-to-lesser without loss of generality, there are only three possible ratios between the excesses, namely, $(a - b) : (b - c)$, $(a - c) : (a - b)$, and $(a - c) : (b - c)$. Now, the ratio $(a - b) : (b - c)$ can be greater than, equal to, or lesser than 1. If the first alternative applies, the ratio can only be the same as $a : b$, $b : c$, or $a : c$, but it is easy to see that the first two cases are equivalent. If the second alternative applies, the ratio $(a - b) : (b - c)$ can only be identical to $a : a = b : b = c : c$. If the third alternative applies, the ratio $(a - c) : (b - c)$ can indeed be the same as any of $c : b$, $b : a$, or $c : a$. This gives $2 + 1 + 3 = 6$ means associated to the first ratio of the excesses. Next, the ratio $(a - c) : (a - b)$ can only be greater than 1: therefore, it can only be the same as $a : b$, $b : c$, or $a : c$, and all three cases may obtain. This gives three additional means. Finally, the ratio $(a - c) : (b - c)$ can only be greater than 1, too: therefore, again, it can only be the same as $a : b$, $b : c$, or $a : c$, but in this case $(a - c) : (b - c) :: a : b$ is idle since it entails $a = b$. This gives two additional means. Thus, there are $6 + 3 + 2 = 11$ means.

The means are eleven but Nicomachus and Pappus only list ten of them. Fortunately, the two lists do not match: each of the two authors describes a mean that is missing in the other. The reason of the omission must lie in the fact that Nicomachus and Pappus relied on different Neo-Pythagorean sources, each of which closed the list as soon as a “holy” decad was completed. Nicomachus can be taken to imply this when he writes “after which the moderns discover four other ‘means’ as well, making up the number ten, which, according to the Pythagorean view, is the most perfect possible” (*Ar.* II.22.1). Following Pappus’ order and including the mean he does not list, the eleven means are defined as follows:

arithmetic	$(a - b) : (b - c) :: a : a$
geometric	$(a - b) : (b - c) :: a : b$
harmonic	$(a - b) : (b - c) :: a : c$
fourth or subcontrary to the harmonic	$(a - b) : (b - c) :: c : a$
fifth or subcontrary to the geometric	$(a - b) : (b - c) :: c : b$
sixth	$(a - b) : (b - c) :: b : a$
seventh	$(a - c) : (a - b) :: b : c$
eighth	$(a - c) : (a - b) :: a : b$
ninth	$(a - c) : (a - b) :: a : c$
tenth	$(a - c) : (b - c) :: b : c$
eleventh	$(a - c) : (b - c) :: a : c$

Pappus, *Coll.* III.47-57, also describes a method for generating all means by starting from the sole “minimal” geometric mean (1,1,1). The method is also described by Theon of Smyrna, *Expositio*, 106-111 Hiller, who draws it from Adrastus, who in his turn used Eratosthenes as his source, by Nicomachus, *Ar.* I.23.6-II.2.2, by Iamblichus, *in Nic.* III.39-45, 112.33-114.29 Vinel, and by Proclus, *in Ti.* II, 18-20 Diehl. Yet, in these authors the method is used to show how to generate successive *geometric* means by starting again from the minimal geometric mean (1,1,1). As a matter of fact, in these authors the method is not used to generate the means, but to generate species-wise the *ratios* of the five basic species (but the two notions are strictly related, as we shall see presently).

The method described by Pappus works as follows. Given a *geometric* mean (c,b,a) with $c < b < a$, one can always take “linear combinations” of the terms c , b , and a to get three new terms C , B , and A . It happens that appropriate linear combinations do give rise to all means. For instance, a new geometric mean is found if we set $C = c$, $B = c + b$, $A = c + 2b + a$. Pappus’ exposition has some gaps, but they can easily be filled; the following table provides a synopsis of the transformations required to generate the indicated mean by taking the associated linear combination of the terms of an assigned *geometric* mean.

arithmetic	$C = c + b, B = c + 2b + a, A = c + 3b + 2a$
geometric	$C = c, B = c + b, A = c + 2b + a$
harmonic	$C = c + b, B = c + 2b, A = c + 3b + 2a$
fourth	$C = c + b, B = c + 2b + 2a, A = c + 3b + 2a$
fifth	$C = c + b, B = c + 2b + a, A = c + 3b + a$
sixth	$C = b + a - c, B = c + 2b + a, A = 2c + 3b + a$
seventh	$C = c, B = b + a, A = c + b + a$
eighth	$C = c + 2b, B = c + 2b + a, A = c + 3b + 2a$
ninth	$C = c + b, B = c + b + a, A = c + 2b + a$
tenth	$C = c, B = c + b, A = c + b + a$
eleventh	$C = b + a, B = 2b + a, A = c + 2b + a$

The reader is urged to keep in mind the second transformation in the list, which is, as said, the only one used by all other sources mentioned above.

To understand how this specific transformation was used to generate all ratios, as Nicomachus does, recall that the standard classification of greater-to-lesser ratios lists five of them, as we learn from instance in Nicomachus, *Ar.* I.17.8-23.3. These are, if reduced to lowest terms and where n , m , and k are integer numbers:

- multiple ratios, of the form $\frac{n}{1} = n$;
- superparticular ratios, of the form $\frac{(n+1)}{n} = 1 + \frac{1}{n}$;

- multiple-superparticular ratios, of the form $k + \frac{1}{n}$, with $k > 1$;
- superpartient ratios, of the form $\frac{(n+m)}{n} = 1 + \frac{m}{n}$, with $1 < m < n$;
- multiple-superpartient ratios, of the form $k + \frac{m}{n}$, with $k > 1$ and $1 < m < n$.

It is clear that this partition of greater-to-lesser ratios is exclusive and exhaustive, for any such ratio can be written as $k + \frac{m}{n}$, with $k \geq 1$ and $0 \leq m < n$.

Now, any geometric mean in lowest terms can be written (b^2, ab, a^2) for suitable integers a and b , with $a > b$, prime to each other.⁹ This amounts to saying that there is a ratio in lowest terms associated to any geometric mean in lowest terms: if the mean is (b^2, ab, a^2) , the ratio is $\frac{a}{b}$,¹⁰ which can be read off within the mean as the two ratios ab/b^2 and a^2/ab ; neither of these ratios, however, can be in lowest terms unless $b = 1$.

With this background material, let us now see what Nicomachus does at *Ar.* I.23.6-II.2.2. As said, he presents a two-arm recursive prescription (πρόσταγμα) for finding the geometric means associated to all kinds of ratios, by starting from the minimal geometric mean of “equality” (1,1,1), which corresponds to the ratio (1,1).¹¹ The prescription maps a mean (z, y, x) , whose terms are called in increasing order “first”, “second”, and “third”, into the mean $(z, z + y, z + 2y + x)$ or, by taking the original terms in reverse order (ἀναστραφέντων) (x, y, z) , into the mean $(x, x + y, x + 2y + z)$. Here is Nicomachus’ text (*IA* I.23.8): πρῶτον πρῶτῳ ἴσον ποιῆσαι, δεύτερον δὲ πρῶτῳ ἅμα καὶ δευτέρῳ, τρίτον δὲ πρῶτῳ καὶ δυσὶ δευτέροις ἅμα καὶ τρίτῳ “make the first equal to the second, the second to the first together with the second, the third to the first and two seconds together with the third”.¹²

Since, as we have just seen, any geometric mean can be written (b^2, ab, a^2) , the prescription can be finally formulated as follows (I skip some parentheses from now on), where b^2 = “first term”, ab = “second term”, a^2 = “third term”:

$$\begin{aligned} b^2, ab, a^2 &\rightarrow b^2, b^2 + ab, b^2 + 2ab + a^2 = b^2, (b + a)b, (b + a)^2 \\ b^2, ab, a^2 &\rightarrow a^2, a^2 + ab, a^2 + 2ab + b^2 = a^2, (a + b)a, (a + b)^2 \end{aligned}$$

⁹ This is proved in *Elem.* VIII.11.

¹⁰ This ratio will also be written (a, b) .

¹¹ The philosophical gist of the procedure lies in the fact that all ratios –that is, the entire unordered world of inequality– is thereby reduced to the unifying principle of equality. In the initial sections of Book II, Nicomachus applies the inverse prescription, thereby getting from any ratio to the ratio of equality. Note that Nicomachus always uses προτάγματα in the plural, apparently taking each of the clauses in the “prescription” as a separate item. I shall stick to the singular to simplify matters.

¹² The absence of the article in the Greek text is a way to formulate the prescription in full generality.

The inverse transformation $^*\rightarrow$ amounts to compounding inversion of extremes and first-arm prescription, as follows:

$$b^2, ab, a^2 \curvearrowright a^2, ab, b^2 \rightarrow a^2, a^2 + ab, a^2 + 2ab + b^2 = a^2, (a+b)a, (a+b)^2$$

Therefore, on account of the above one-to-one map between means in lowest terms and ratios in lowest terms, the same two-arm prescription applied to ratios reads:

$$(a,b) \rightarrow (b+a,b), \text{ that is, } \frac{a}{b} \rightarrow \frac{(b+a)}{b} = 1 + \frac{a}{b}$$

$$(a,b) ^*\rightarrow (a+b,a), \text{ that is, } \frac{a}{b} \rightarrow \frac{(a+b)}{a} = 1 + \frac{b}{a}$$

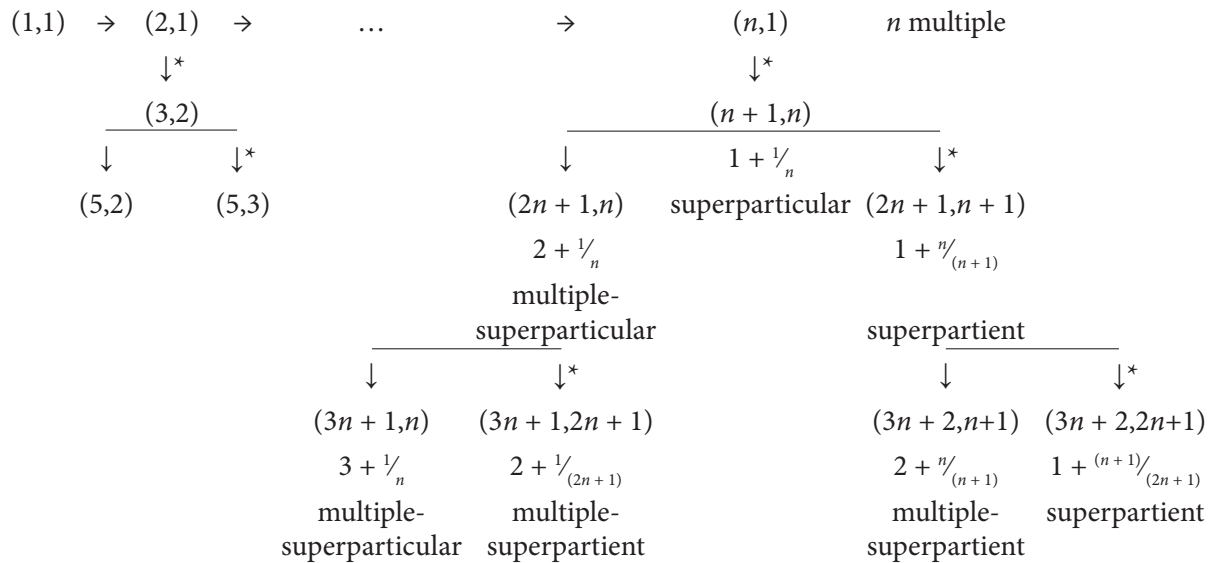
For example, take the mean (1,1,1) and apply the prescription once; both arms give the same result: $(1,1,1) \rightarrow (1,1+1,1+2+1) = (1,2,4)$, which is the double ratio. Let us iterate the prescription by applying it to the double ratio itself:

$$(1,2,4) \rightarrow (1,1+2,1+4+4) = (1,3,9)$$

$$(1,2,4) \curvearrowright (4,2,1) \rightarrow (4,4+2,4+4+1) = (4,6,9)$$

The former is the triple ratio, the latter is the sesquialter, or hemiolic, ratio

Thus, the “direct” ($\delta\rho\theta\tilde{\omega}\zeta$) arm sends ratios of kind X into ratios of kind multiple-X because at every step it adds one unit to the original ratio, eventually keeping the resulting ratios within such a multiple-X kind; the “inverse” arm of the prescription sends ratios of kind X into ratios of a different kind because at every step it adds one unit to the *inverse* of the original ratio, eventually transforming it into a superpartient or multiple-superpartient ratio. All and only the five kinds of ratios are generated through this procedure, as can be seen from a rough flow chart that summarizes the entire process.



Let us note, finally, that, in its domain of application, the “direct” arm sends a ratio into a greater ratio (this is obvious, since it simply adds 1 to the original ratio), whereas the “inverse” arm may send a ratio either into a greater or into a smaller ratio (just check in the flow chart above).

3. A Concise Statement by Nicomachus: in Fact, a Gloss

Let us have a close look at the prescription:

$$b^2, ab, a^2 \rightarrow b^2, b^2 + ab, b^2 + 2ab + a^2 = b^2, (b + a)b, (b + a)^2$$

$$b^2, ab, a^2 \star \rightarrow a^2, a^2 + ab, a^2 + 2ab + b^2 = a^2, (a + b)a, (a + b)^2$$

It is evident that the extremes of all the involved means are square numbers, that the largest terms of the two means in the right-hand side are the same and a square, namely, $(b + a)^2 = (a + b)^2$, and that the largest term of the original mean becomes the smallest term of the mean that results from the inverse transformation. Moreover, if we write the inverse transformation in terms of the direct transformation as done above, namely

$$b^2, ab, a^2 \curvearrowright a^2, ab, b^2 \rightarrow a^2, a^2 + ab, a^2 + 2ab + b^2 = a^2, (a + b)a, (a + b)^2$$

and we skip the inversion step $\curvearrowright$, thus:

$$a^2, ab, b^2 \rightarrow a^2, a^2 + ab, a^2 + 2ab + b^2 = a^2, (a + b)a, (a + b)^2$$

we see that in both arms of the prescription the smallest term in the mean remains the same under the transformation: in the direct arm, it is b^2 , in the inverse arm, it is a^2 .

Nicomachus also seems to have noticed some of these properties, for at *Ar. I.23.15* –in the middle of a series of examples– he drops in the following remark:¹³ ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν καὶ ἀφ’ ἧς ἀμφοτέραι, ὁ μὲν ἔσχατος τετράγωνος ὁ αὐτὸς μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι “In the case of all these <relations> that are <thus> differentiated, and of the one from which both <of the differentiated ones are derived>, the last square remains the same; the first <term> is transformed into the smallest, and invariably the extremes are squares”.

The problem with this statement is that it is too concise, fairly ambiguous and under-determined by the context (there is no mention of ἔσχατος term in what precedes,¹⁴ nor of διαζευχθεῖσαι relations; the qualifiers “first” and “smallest” do not belong to homogeneous descriptive levels; the sense in which “the first <term> is transformed into the smallest” is hardly clear, and can only apply to one of the two arms of the prescription, not to both; the statement “the last square remains the same” can instead be taken to apply to both arms,

¹³ This remark makes the entire section *Ar. I.23.15* in Hoche’s edition.

¹⁴ The adjective will only occur at *Ar. II.2.2* in the present context.

as seen above, or –more straightforwardly– to the direct arm only), and so badly formulated (the same set of objects is first qualified by *πασῶν* and then by *ἀμφοτέραι*) as to be unsyntactical (the presence of *ἀφ' ἧς* is jarring). An exegetical problem resulted, which was “solved” in manifold ways by the commentators of *Ar.* and that even interfered with the treatise's text in most manuscripts. I have written “solved” between quotation marks because some of the “solutions” set forth by the commentators are weird or irremediably wrong. Even D'Ooge has a partly incorrect translation, since he writes “the last term is always the same and a square”, which is definitely not what the Greek text says.

As a partial excuse for the performances of the commentators, let me state clearly that I think that this sentence –despite being witnessed to by the entire manuscript tradition– is a gloss. First and foremost, this is a question of style: there is not one single passage in the entire *Ar.* that is so clumsily written and so cryptic, and which disrupts the line of argument in so offensive a way. I also surmise that the origin of the gloss lies in a remark by Iamblichus, where he is just commenting on what Nicomachus does: *ἐπὶ δὲ πασῶν τῶν πλασσομένων σχέσεων καὶ ἀφ' ὧν αἱ πλάσεις, οἱ ἄκροι τετράγωνοι γίνονται*, says Iamblichus.¹⁵ And what he says is crystal-clear, and perfectly syntactical Greek. Someone after him thought it better to buttress Nicomachus' text –possibly only in the margin of his own manuscript exemplar of *Ar.*– with an enriched version of Iamblichus' statement. The final outcome of this brilliant idea we read in our manuscripts.

4. Philological Context

The task of editing the *Introductio arithmetica* is made particularly difficult by the fact that it was, after its composition in the 2nd century, the most widely used textbook of number theory for more than one millennium. For this reason, it was frequently copied: we know about 100 manuscript witnesses of it, of which nearly 90 contain the complete work and the rest fragments or *excerpta*.¹⁶ Not only this: there is not the slightest doubt – as we shall see *ad abundantiam*– that the text has undergone modifications, and that wild contamination has occurred since the earliest witnesses we have access to; we have all reasons to think that this also occurred at a pre-traditional stage. Anyway, the only edi-

¹⁵ At *in Nic.* III.68, 120.9-10 Vinel. I add that, had Iamblichus read section I.23.15 in his Nicomachus, he would hardly have omitted elaborating on *all* claims contained in the sentence.

¹⁶ A fairly complete list of manuscripts is in W. Haase, *Untersuchungen zu Nikomachos von Gerasa*, Dissertation, Eberhard-Karls-Universität Tübingen 1982, 319-398. The database *pinakes* lists 107 items, but with some misattributions; see also M. L. D'Ooge – F. E. Robbins – L. Ch. Karpinski (transl. comm.), *Nicomachus of Gerasa, Introduction to Arithmetic*, Ann Arbor 1926, 146-166, for a list of manuscripts and some, very partial and preliminary, investigations into a part of the manuscript tradition. My list is found in the Appendix.

tion of *Ar.* is Richard Hoche's,¹⁷ who used a random and quite limited sample of manuscripts, namely, those readily available to him because they were kept in nearby German libraries. Hoche took the one carried by Gott. philol. 66 as his text of reference, relying on a wrong dating of the manuscript to 10th century.¹⁸

The standard view of the exegetical record on *Ar.* is that one ancient commentary proved so successful that we have four different versions of it, referred to as Recensions I-IV.¹⁹ Detail is as follows. The Neo-Platonic philosopher Ammonius held a course on *Ar.*²⁰ whose lecture notes were published by his pupils. The edited notes have the form of a running commentary organized in "lemmas", which clarify single statements in the text.²¹ As a matter of fact, the comments on the technical portions of *Ar.* frequently feature lists of numerical examples (ratios, figured numbers, means, etc.), exactly as *Ar.* does. This happens in general in mathematical and astronomical commentaries of Late Antiquity, which mainly comprise explanations of the *technologia*, examples, scholarly and antiquarian digressions.²²

Every Recension is divided in two Books, as *Ar.* is. Rec. I, which both the manuscripts and modern scholarship assign to John Philoponus, was edited in the 1860s by Hoche in a series of three *Schulprogramme*.²³

¹⁷ *Nicomachi Geraseni pythagorei Introductionis arithmeticae libri II*, Lipsiae 1866.

¹⁸ It was in fact penned only in the late 13th century, all the while using an imitative script: G. De Gregorio – G. Prato, "Scrittura arcaizzante in codici profani e sacri della prima età paleologa", *Römische Historische Mitteilungen* 45 (2003), 59-101; and, most recently, F. Acerbi – A. Gioffreda, "Manoscritti scientifici della prima età paleologa in scrittura arcaizzante", *Scripta* 12 (2019), 9-52: 26 and 36 for the dating to late 13th century.

¹⁹ P. Tannery, "Rapport sur une mission en Italie du 24 Janvier au 24 Février 1886", *Archives et Missions scientifiques et littéraires*, 3^e série 13 (1888), 409-455, reprinted in Id., *Mémoires Scientifiques*, II, Toulouse – Paris 1912, 269-331: 302-310. See also L. G. Westerink, "Deux commentaires sur Nicomaque: Asclépius et Jean Philopon", *REG* 82 (1968), 526-535; and L. Tarán, *Asclepius of Tralles, Commentary to Nicomachus' Introduction to Arithmetic*, Transactions of the American Philosophical Society 59.4, Philadelphia 1969, 5-20, in particular as to the relations between Rec. I and Rec. III.

²⁰ For a first orientation on Ammonius' school and on the main characters mentioned below, see the entries in R. Goulet (ed.), *Dictionnaire des philosophes antiques*, 7 vol., Paris 1994-2018.

²¹ On the structure of the Neoplatonic commentaries, see the synthesis by Ph. Hoffmann, "What was Commentary in Late Antiquity? The Example of the Neoplatonic Commentators", in M. L. Gill – P. Pellegrin (eds.), *A Companion to Ancient Philosophy*, Malden (Mass.) 2006, 597-622. On the way these lecture notes were published, see the seminal M. Richard, "ΑΠΟ ΦΩΝΗΣ", *Byzantion* 20 (1950), 191-222.

²² For the exegetical practices of Eutocius, one of Ammonius' pupils, see most recently F. Acerbi, "Commentari, scolii e annotazioni marginali ai trattati matematici greci", *S&T* 10 (2012), 135-216. with bibliography.

²³ For a *specimen novae editionis criticae*, unfortunately not achieved, see Haase, *Untersuchungen* (cit. n. 16), 401-447.

Hoche himself, when editing Rec. I, listed the differences between Rec. I and Rec. II for the first book of *Ar.* in the preface of his 1865 *Schulprogramm*;²⁴ in doing so, he relied on exactly one manuscript, namely, Zeitz, Stiftsbibliothek 67. Hoche never published a similar list for the scholia to the second book. Almost a century later, A. Delatte edited the Rec. II scholia for Book II, using Vat. gr. 1411 and Athens, EBE 1238, as witnesses.²⁵ All witnesses of Rec. II anchor the scholia on the text by means of numerals; this graphic convention, despite being found in Hoche's edition, is only adopted by a small number of witnesses of Rec. I. In the manuscripts, Rec. II is duly ascribed to Philoponus. More on Rec. II in Section 6 below.

Rec. III is available in a critical edition.²⁶ Manuscripts and scholars concur in ascribing it to Asclepius of Tralles, a pupil of Ammonius and possibly a schoolmate of Philoponus.

As we shall see in Section 6,²⁷ Rec. IV is not a recension of Ammonius' lecture notes²⁸ but an independent commentary, to be assigned with certainty to the Byzantine period because of its prolixity and of its paraphrastic structure, to such an extent that large portions of it offer a self-contained argument.

The pattern of association of *Ar.* with its several commentaries is quite complex: there are about 40 manuscripts in which one of the Recensions is copied as a self-contained text (either following or preceding the Nicomachean treatise, or even in absence of it), about 20 manuscripts in which the Recension lies in the margins. About 40 manuscripts carry *Ar.* with or without fully-fledged commentaries in form of scholia; many of these have scholia unrelated to any of the Recensions.

²⁴ Ἰωάννου γραμματικοῦ Ἀλεξανδρέως (τοῦ Φιλοπόνου) εἰς τὸ πρῶτον τῆς Νικομάχου ἀριθμητικῆς εἰσαγωγῆς, ed. R. Hoche, 2 fasc., Gymnasium zu Wesel 1864-65, II-xv. This edition is totally unreliable, as we shall see.

²⁵ A. Delatte, *Anecdota Atheniensia et alia. Tome II. Textes grecs relatifs à l'histoire des sciences*, Liège – Paris 1939, 129-187.

²⁶ Tarán, *Asclepius* (cit. n. 19), who based himself on Ambros. B 77 sup., ff. 102r-149v, and Monac. gr. 431, ff. 98r-114v (ca. 1330; *Diktyon* 44879), a copy of which is Par. gr. 2376, ff. 1r-56r. The "Notes to the Text" in Tarán's edition, 73-81, give a fairly clear idea of aims, methods and philosophical stand of Rec. I and III.

²⁷ See the discussion of the entire textual tradition in F. Acerbi, "La tradition manuscrite de la « Recension IV » du commentaire à l'*Introductio arithmetica* de Nicomaque", submitted. This tradition is split into two branches, whose ancestors are extant manuscripts.

²⁸ The same assessment is in Tarán, *Asclepius* (cit. n. 19), 20. As a matter of fact, it is enough to read Rec. IV to grasp the difference from the other Recensions. Ammonius is never referred to in Rec. IV.

5. Texts

The first text I present below is of course Nicomachus, *Ar.* I.23.15, in all manuscripts I was able to check and that do contain the end of Book I (82 items). I shall partition the textual record into five groups, according to distinctive variant readings.

The exegetical record comprises the following items:

- 1) Ammonius' lecture notes, Rec. I (Philoponus) in a limited set of manuscripts.
- 2) Ammonius' lecture notes, Rec. II (anonymous), in the text of most of the extant manuscripts.
- 3) Ammonius' lecture notes, Rec. III (Asclepius), in Tarán's edition.
- 4) The anonymous Byzantine commentary called "Recension IV", in my own critical text.²⁹
- 5) The Adjunct to the text of the *Introductio arithmetica* in the same manuscripts as those used for *Ar.* I.23.15 (this is subsection 5.1), and a scholium thereon (subsection 5.2). I shall partition the textual record of the Adjunct into six groups, according to distinctive variant readings.
- 6) The scholia in Matrit. 4678 and other manuscripts (with glosses).
- 7) Glosses and scholia in Monac. gr. 482 and Laur. Plut. 28.35.
- 8) Four scholia in Ambros. G 62 sup. and other manuscripts; the scholia are ascribed to Michael of Ephesus, to an anonymous from the town of Δυρράχιον (present-day Durrës), to Eustratius of Nicaea, and to a judge Nicholas Dishypatos, respectively.
- 9) Four scholia in Vat. gr. 256 and other manuscripts.
- 10) Glosses and scholia by Isaak Argyros in Norimb. Cent. V app. 36.³⁰
- 11) A scholium eventually included in some Rec. I manuscripts and ascribed in most of them to some Theodoros/Demetrios Protocensor.³¹
- 12) Glosses, and scholia to the Adjunct, in Hamburg, S-UB, philol. 88, Vat. gr. 2387, and related manuscripts.
- 13) Dishypatos' and the anonymous from Dirrachios' (*sic*) scholia in Vat. gr. 198 and in a copy of it.
- 14) Glosses and scholia in Marc. gr. Z. 320 and related manuscripts.
- 15) Glosses and a scholium to the Adjunct by a later hand in Par. gr. 2373.
- 16) Glosses and scholia in Kharkow, UL, 269-p, 369-c and related manuscripts.
- 17) Glosses and scholia in Marc. gr. Z. 316 and 319.
- 18) Glosses and a scholium in Laur. Conv. Soppr. 30.

²⁹ Acerbi, "La tradition" (cit. n. 27).

³⁰ First edited, with some misreadings, in C. F. A. Nobbe, *Codicum Quelferbytani et Norimbergensis scholia graeca ad librum I. Isagoges Nicomacheae nunc primum edita*, Lipsiae 1862, 18.

³¹ It is partly edited by Hoche, Ἰωάννου (cit. n. 24), xiv-xv, who calls the author Theodoros.

In my editions, I regularize punctuation and accents, and neglect variants of spelling such as movable *ny*, –σσ– vs. –ττ–, geminate or simple *ny*, etc. Numerals are transcribed without apex, ordinals carry the termination at the exponent. I frequently omit the anchoring lemma of the scholia; otherwise, it is within angular brackets <*>. Each textual unit comprises the list of the manuscripts used, the text itself, and the critical apparatus, which is always negative. The variant readings recorded within the text are keyed to the witnesses by means of the number that characterizes the latter's shelfmark. The infra-linear glosses are inserted in the text, within braces. Other conventions are explained in due course. A complete list of the manuscripts employed, with additional information on each of them, is set out in the Appendix.

NICOMACHUS, AR. I.23.15, IN THE MANUSCRIPTS

The textual record is partitioned into five groups, according to distinctive variant readings; these are marked in red; the first text has no such marking. Singular readings are recorded in the apparatus; manuscripts marked by the same colour share interesting singular readings.

A

Ambros. G 62 sup., f. 43r; Ambros. P 121 sup., f. 130v; [Bucur., BAR gr. 620, pp. 220-221](#); [Gott. philol. 66, ff. 138v-139r](#); Guelf. 36 Gud. gr., f. 17r-v; Hamburg, S-UB, philol. 89, p. 75; Laur. Conv. Soppr. 30, f. 42v; Laur. Plut. 28.35, f. 31r; Laur. Plut. 58.29, f. 152v; Leiden, BRU, Voss. gr. Q^o 23, ff. 29v-30r; Marc. gr. Z. 309, f. 193r; Marc. gr. Z. 316, f. 60v; Marc. gr. Z. 318, f. 20v; Marc. gr. Z. 320, f. 52r-v; Marc. gr. Z. 333, f. 61r; Marc. gr. Z. 514, f. 24v; Marc. gr. Z. 592, f. 36v; Matrit. 4678, f. 25v; Monac. gr. 482, f. 132v; Mutin. α.T.8.14, f. 27r-v; Mutin. α.U.9.7, f. 38r; Neapol. III.C.1, ff. 19v-20r; Norimb. Cent. V app. 36, f. 18r; Oxon. Bodl. Holkham 71, f. 278r-v; Oxon. Bodl. Laud. gr. 44, p. 89; Par. Coislin 174, f. 58r; Par. gr. 2107, f. 84v; Par. gr. 2374, f. 21r-v; Par. gr. 2376, f. 78r; Par. gr. 2450, f. 111r; Par. gr. 2479, f. 91v; [Par. gr. 2480, pp. 339-340](#); Par. gr. 2531, f. 80r-v; Par. suppl. gr. 450, ff. 34v-35r; Roma, Arch. S. Paolo 24C, f. 53v; Scorial. Σ.II.15, f. 171r-v; Scorial. Υ.III.12, f. 20v; Scorial. X.I.9, ff. 46v-47r; Vat. Barb. gr. 273, f. 122r; Vat. gr. 186, f. 243r; Vat. gr. 195, f. 42v; Vat. gr. 198, f. 19r; Vat. gr. 1026, f. 63v; Vindob. phil. gr. 62, f. 24r; Vindob. phil. gr. 220, f. 61r-v

ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν καὶ ἀφ' ἧς ἀμφοτέραι, ὁ μὲν ἔσχατος τετράγωνος ὁ αὐτὸς μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

Ambros. P 121 sup.: ἄκρως scr. sed –οι s.l. m.1 | [Bucur., BAR gr. 620: ἀμφοτέρα](#) τετράγωνος ὁ αὐτὸς μένει | [Gott. philol. 66: ἀμφοτέρα](#) | Laur. Plut. 58.29: πάντως] [s.l. γρ. πάντες](#) | Leiden, BRU, Voss. gr. Q^o 23: ἀμφοτέρων | Marc. gr. Z. 309: ἀμφοτέροι | Marc. gr. Z. 514: τετράγωνος ἐσχάτως μένει (marg. γρ. ὡσαύτως) | Neapol. III.C.1: –γον– (pr.) | Par. gr. 2374: διαζευχθεισῶν σχέσεων

| Par. gr. 2480: ἀμφοτέρα | Par. gr. 2531: ἀμφοτέρων | Scorial. Υ.ΙΙΙ.12: πάντων | τὸ | om. πάντως
δὲ οἱ ἄκροι τετράγωνοι | Vat. gr. 1026: ἔσχατος ἀριθμὸς τετράγωνος

B

Ambros. I 8 sup., f. 33v; Athens, EBE 1115, p. 58; Leiden, BRU, Periz. Q^o 39, f. 23r; Marc. gr. Z. 317, f. 23r; Marc. gr. Z. 319, f. 38v; Marc. gr. Z. 330, ff. 301v-302r; Marc. gr. Z. 595, f. 32r; Neapol. III.C.6, f. 31v; Oxon. Bodl. Selden Supra 20, f. 16v; Par. gr. 2372, f. 25v; Par. gr. 2373, f. 17r; Par. gr. 2762, ff. 40v-41r; Vat. gr. 196, ff. 14v-15r; Vat. gr. 256, f. 316r; Vat. gr. 1040, f. 20r; Vat. Ottob. gr. 310, f. 142r

ἐπὶ πασῶν δὲ τῶν διαzeugθειςῶν καὶ ἀφ' ἧς ἀμφοτέραι, ὁ μὲν ἔσχατος τετράγωνος **ὡσαύ-
τως** μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

Ambros. I 8 sup.: ὡσαύτως] ὁ αὐτὸς s.l. | Athens, EBE 1115 & Vat. gr. 256: ὡσαύτως] γρ. καὶ ὁ
αὐτὸς s.l. | Marc. gr. Z. 319: πάντες | Marc. gr. Z. 317: διαλεχθειςῶν καὶ διαzeugθειςῶν | Marc.
gr. Z. 595 & Par. gr. 2762: διαλεχθειςῶν sed -ζευ- s.l. | ὁ (pr.)] ἡ | Par. gr. 2372: ἀμφοτέρα

C

Bonon BU 2263, f. 16r-v; Hamburg, S-UB, philol. 88, ff. 23v-24r; Mutin. α.W.3.1, f. 18r; Vat. gr. 2387, f. 48v

ἐπὶ πασῶν δὲ τῶν διαzeugθειςῶν καὶ ἀφ' ἧς ἀμφοτέραι, ὁ μὲν ἔσχατος τετράγωνος **{om. ὁ
αὐτὸς}** μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

Mutin. α.W.3.1: μεταβαίνειν

D

Ambros. B 77 sup., f. 57v; Berol. Phillipps 1549, f. 84r; Bucur., BAR gr. 520, p. 118; Cizensis 67, f. 51v; Erlangen, UL A.8, f. 38v; Leiden, BRU, BPG 74G, f. 24v; Monac. gr. 76, f. 247v; Oxon. Lincoln gr. 33, f. 109v; Par. gr. 2063, f. 33r; Par. gr. 2377, f. 84r; Vat. gr. 197, f. 15v; Vat. gr. 1411, f. 76r

ἐπὶ πασῶν δὲ τῶν διαzeugθειςῶν καὶ ἀφ' ἧς ἀμφοτέραι, ὁ μὲν ἔσχατος τετράγωνος ὁ
αὐτὸς μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει, **πάντες** δὲ οἱ ἄκροι τετράγωνοι.

Monac. gr. 76: τετράγωνοι] -ος | Par. gr. 2063: δ' οἱ | Par. gr. 2377: ἀμφοτέραι] -ων s.l. | πάντες]
-ω- s.l. | Berol. Phillipps 1549 & Par. gr. 2377: **marg. ἐν ἄλλοις οὕτως· ὁ μὲν πρῶτος τετράγωνος
ὁ αὐτὸς μένει, ὁ δὲ ἔσχατος εἰς τὸν ἐλάττονα μεταβαίνει** | Erlangen, UL A.8: om. τὸν

E

Kharkow, UL, 269-p, 369-c, f. 87v; Oxon. New Coll. 299, f. 55v; Par. gr. 2481, f. 18v; Par. gr. 2483, p. 152; Scorial. P.II.3, f. 431v; Vat. gr. 1051, f. 20v

ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν καὶ ἀφ' ἧς ἀμφότεραι, ὁ μὲν ἔσχατος τετράγωνος ὦν αὐτὸς μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

Kharkow, UL, 269-p, 369-c & Par. gr. 2483: marg. ἐν ἄλλοις οὕτως· ὁ μὲν πρῶτος τετράγωνος ὁ αὐτὸς μένει, ὁ δὲ ἔσχατος εἰς τὸν ἐλάττονα μεταβαίνει | Par. gr. 2483: s.l. (ἀμφο)τέρων m. rec. | Vat. gr. 1051: ἔσχατος s.l. | Scorial. P.II.3: ἀμφοτέρων

Glosses to Ar. I.23.15

Vat. gr. 195, f. 42v

ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν καὶ ἀφ' ἧς ἀμφότεραι, ὁ μὲν ἔσχατος τετράγωνος ὁ αὐτὸς μένει, ὁ δὲ πρῶτος {λόγος δηλονότι δ ἢ ὁ θ} εἰς τὸν ἐλάττονα {** ἐπίτριτος ** ὁ ι** ἐπίτριτος} μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

COMMENTARIES

C.1. Recension I

In Hoche's edition, this is sect. I.ρς, but the numbering has almost no manuscript authority. Singular readings are recorded within the text; recurrent clusters of witnesses are identified by the same colour. A detailed comparison of Texts C.1-4 will be carried out in Section 6.2. The first three manuscripts contain only Rec. I.

Ambros. I 83 inf., f. 89r-v (m. rec.); Angel. gr. 1, f. 33r-v; BNCF, Fondo Naz. (*olim* Magliab.) II.III.37, f. 46r-v; Bucur., BAR gr. 620, pp. 220-222; Gott. philol. 66, ff. 138v-139v; Hamburg, S-UB, philol. 89, pp. 368-370; Laur. Plut. 28.35, f. 29r; Marc. gr. Z. 333, f. 61v; Monac. gr. 76, ff. 158v-159r; Monac. gr. 482, ff. 218v-219r; Mutin. α.T.8.14, f. 27v; Mutin. α.U.9.7, f. 38r-v; Par. Coislin 174, f. 102v; Par. gr. 2480, pp. 339-341; Par. gr. 2483, pp. 145-146; Par. gr. 2531, f. 79v; Roma, Arch. S. Paolo 24C, ff. 53v-54r; Scorial. P.II.3, ff. 431v-432r, 432v-433r; Vat. gr. 198, f. 19r

ρς. διαζευχθεῖσαι εἰσι σχέσεις αἱ τε {τε om. 28.35 482} κατ' ὀρθὸν ἐπιζευγνύμεναι {ἐπε- 620} καὶ αἱ κατὰ ἀναστροφὴν συμπλεκόμεναι· διέζευκται γὰρ τοῦ ὀρθοῦ τὸ ἀνεστραμμένον {ἀνα- 76}· ἐκάστης μὲν οὖν σχέσεως οἱ ἄκροι πάντως εἰσὶ τετράγωνοι {τ. ε. 28.35 482 : τεταγμένοι sed corr. m.1 1 : ἐπιτετράγωνοι 2531}, δι' ἣν αἰτίαν ἐροῦμεν {ἐ. α. 89 2483 P.II.3 198}· ἀλλ' {ὅκι 83} ὁ μὲν ἔσχατος ὁ αὐτὸς μένει {ὡς αὐτὸς P.II.3}, φησὶν· οὐ πρόεισι γὰρ {γὰρ om. P.II.3} ἐφ' {εἰς 620} ἕτερον ἢ μειούμενος {μειζούμενος P.II.3} ἢ αὐξάμενος {-όμε- 83 37}, ἔσχατος ὦν· ὁ δὲ πρῶτος ἐκ τῆς ἀναστροφῆς πρὸς τὸν ἐλάττονα πρόεισι μέχρι μονάδος, ἐπεὶ {ἐπὶ 83} καὶ {ἀεὶ add. 2531} ἀπὸ {τῆς s.l. 37} μονάδος {ἐπεὶ — μονάδος om. 76} ἤρξατο. τοῦτο δὲ δῆλον ἐκ τε τῶν {τῶν om. P.II.3} προειληφειῶν σχέσεων καὶ {καὶ ἐκ 83 37 89 2483 198 : καὶ 333 : κὰν 76} τῶν {καὶ αὐτῶν P.II.3} λέγεσθαι μελλουσῶν. πάντως {πάντων 89 2483 198} δέ, φησὶν {φησὶν om. 2483 quod lemma fecit}, οἱ ἄκροι τετράγωνοι· {πάντως — τετράγωνοι om. P.II.3}

sed perperam incl. 6a} ἐπὶ μὲν {ακτῶν add. P.II.3} ἡμιολίων, τοῦ δ καὶ ς {δὶς 2531} καὶ θ {τοῦ δ ιβ ις 83 37}, ὁ δ καὶ ὁ {ὁ om. 83 37 333 2483 P.II.3} θ· ἐπὶ δὲ τῶν ἐπιτρίτων, τοῦ θ καὶ ιβ καὶ ις, ὁ θ καὶ ὁ ις {τοῦ θ ιβ ις, ὁ θ καὶ ις 333 : καὶ ὁ ις s.l. m. rec. α.T.8.14 : ὁ θ καὶ ις 83 620 37}, καὶ ἐπὶ πάντων ὁμοίως· καὶ τοῦτο εὐλόγως {ἀλόγως 2483 : ἀναλόγως P.II.3}· ἐδείχθη γὰρ παρὰ τῷ γεωμέτρῃ ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι {ἐλαττον 2531} ᾧσι πρὸς ἀλλήλους τὸν αὐτὸν λόγον ἔχοντες, οἱ ἄκροι αὐτῶν τετράγωνοί εἰσιν. ἐπεὶ οὖν ἐπὶ τῶν ἡμιολίων ὁ δ καὶ ὁ ς καὶ ὁ {ὁ om. 333} θ ἐλάχιστοι πρὸς ἀλλήλους εἰσὶ {ε. π. α. 620} καὶ τὸν αὐτὸν ἔχουσι λόγον {λ. ἔ. 28.35 482}, διὰ τοῦτο οἱ ἄκροι τούτων τετράγωνοί εἰσιν. ἀλλ' ἴσως εἴποι τις ἄν· {ὅτι γε add. 333 : s.l. m. rec. α.T.8.14} οὐκ εἰσὶν ἐλάχιστοι {οὔτοι add. 333 : s.l. m. rec. α.T.8.14}· οὐδὲ γάρ εἰσι πυθμένες τῶν ἡμιολίων· ἰδοῦ γάρ, ὁ γ {τρίτος 2483 P.II.3} τοῦ β ἡμιολίου ἐστίν. πρὸς δ λέγομεν ὅτι δύο μὲν ἐφεξῆς ἔσχατοι ἡμιολίοι εὐρίσκονται, ὡς τὰ β καὶ τὰ {τὰ om. 333} γ, τρεῖς δὲ ἐφεξῆς {ἔσχατοι — ἐφεξῆς om. α.U.9.7 2483 P.II.3} ἐλάττονες τοῦ δ καὶ {τοῦ add. 28.35} ς καὶ τοῦ {τοῦ om. 1 620 89 76 174 2483 P.II.3 198} θ {τοῦ δ ς θ 333} οὐδέποτε. ὡσαύτως {δὲ add. 28.35 482} καὶ δύο μὲν ἐπιτρίτους, τὸν γ καὶ τὸν δ, εὐρεῖν ἔστι, τρεῖς δὲ ἐφεξῆς ὑποκάτω τοῦ θ καὶ τοῦ {τοῦ om. 83 37} ιβ καὶ τοῦ ις ἀδύνατον. καὶ ἐπὶ τῶν ἄλλων ὡσαύτως· εἰκότως οὖν οὔτοι οὐ {οὔτοι καὶ 89 198 : οὐ om. 76 174 333 : οὐ del. m. rec. α.T.8.14} τοὺς ἄκρους τετραγώνους ἔχουσιν.

Marc. gr. Z. 333: ἐκ τῶν ἐπιμερῶν κατ' εὐθείαν ληφθέντων γεννῶνται οἱ πολλαπλασιεπιμερεῖς, ἀντεστραμμένως δὲ οἱ ἑτερογενεῖς ἐπιμερεῖς, ἥγουν ἑτεροειδεῖς· τὸ γὰρ ἐπιδιμερὲς καὶ τὸ ἐπιτριμερὲς καὶ τὸ ἐπιδίτριτον καὶ τὸ ἐπιπενταέβδομον ὁμογενῆ μὲν εἰσιν ὡς πάντα ἐπιμερῇ, ἑτεροειδῇ δέ· ἕτερον γάρ ἐστι τὸ ἐπιδιμερὲς τοῦ ἐπιτριμεροῦς, καὶ τοῦτο τοῦ ἐπιτετραμεροῦς extr. e Rec. IV add. in fine

C.2. Recension II

The text displays substantial variants Hoche did not record; they are shared by all witnesses unless otherwise stated. Singular readings are recorded within the text; recurrent clusters of witnesses are identified by the same colour.

Berol. Phillipps 1549, ff. 84v-88r; Cizensis 67, ff. 35v-36r; Oxon. Lincoln gr. 33, f. 110r; Par. gr. 2377, f. 87v-88r; Scorial. Υ.I.12, f. 45r-v; Vat. gr. 1411, f. 60r

<ρ>νζ. διαζευχθεῖσαι σχέσεις εἰσὶν αἱ τε κατ' ὀρθότητα τῶν προτέρων ἐκκειμένων ἀπογεννώμεναι καὶ αἱ {αἱ om. 67} κατὰ ἀναστροφὴν· διέζευκται γὰρ τοῦ ὀρθοῦ τὸ ἀντεστραμμένον· ἐφ' ἐκάστης οὖν φησι σχέσεως οἱ ἄκροι πάντως {-ες 1549 2377} εἰσὶ τετράγωνοι, δι' ἣν αἰτίαν ἐροῦμεν· ἀλλ' ὁ μὲν ἔσχατος, τουτέστιν ὁ ὑπόλογος ἐν πάσαις ταῖς ἐκθέσεσι ταῖς τε κατ' ὀρθότητα ἐκκειμέναις ταῖς τε κατ' ἀντιστροφὴν, τετράγωνος ὢν ὁ αὐτὸς μένει, τουτέστιν ὑπόλογος. οἷον ἐν ἡμιολίῳ σχέσει ὀρθῶς ἐκκειμένη {-μενοι

67} ἦσαν ὁ δ ς θ, καὶ πάντως ὑπόλογος ἦν ἐν τούτοις ὁ δ (τοῦτον γὰρ φησιν ἔσχατον). ἀντιστραφέντες {-στρέφοντες 33} δ' ἦσαν οὕτως θ ς δ, καὶ πάλιν ὁ δ, εἰ καὶ ὕστερος τέτακται, ἀλλὰ πάλιν ὑπόλογός ἐστι πρὸς τὸν ς· ὁ δὲ πρῶτος, ἡγουν {ἦτοι 33} ὁ μέγιστος πρόλογος, εἰ ἀπὸ τούτου ἄρχοιτο δηλαδὴ {δηλονότι 33} ἡ τῆς ἐξῆς σχέσεως πρόβασις, ὑπόλογος γίνεται ἐν ἐκείνῃ. οἷον ἔστω ἡ ῥηθεῖσα ἡμιόλιος σχέσις ἀντεστραμμένη, ἡγουν {οἶον 33} θ ς δ· ἐκ ταύτης γίνεται ἄλλη σχέσις θ ιε κε. φανερόν οὖν ὅτι ἐν μὲν ἐκείνοις πρόλογος ἦν ὁ θ καὶ μέγιστος {-ον 1549 2377 Υ.Ι.12 1411}, ἐν δὲ τούτοις καὶ ὑπόλογος καὶ ἐλάχιστος. πάντες δέ, φησὶν, οἱ ἄκροι τετράγωνοι· ἐπὶ μὲν γὰρ τῶν ἡμιολίων, τοῦ δ καὶ ς καὶ θ {ὁ δ ς θ 33}, ὁ δ καὶ ὁ θ τετράγωνοι {-ος 67}· ἐπὶ δὲ τῶν ἐπιτρίτων, τοῦ θ καὶ ιβ καὶ ις {θ ιβ ις 33}, ὁ θ καὶ ὁ {ὁ om. 1549 2377} ις, καὶ ἐπὶ τῶν ἄλλων ὁμοίως· καὶ τοῦτο εἰκότως· ἐδείχθη γὰρ παρὰ τῷ γεωμέτρῃ ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ὥσι τῶν τὸν αὐτὸν λόγον ἔχόντων αὐτοῖς, οἱ ἄκροι αὐτῶν τετράγωνοί εἰσιν. ἐπεὶ οὖν ἐπὶ τῶν ἡμιολίων ὁ δ, ὁ ς καὶ ὁ θ {δ ς θ 33} ἐλάχιστοί εἰσι πρὸς ἀλλήλους τῶν τὸν αὐτὸν λόγον ἔχόντων αὐτοῖς (οὐδὲ γὰρ ἐστὶν εὐρεῖν πρὸ αὐτῶν ἡμιολίους τρεῖς ἐφεξῆς), διὰ τοῦτο οἱ ἄκροι αὐτῶν τετράγωνοί εἰσι. ἀλλ' ἴσως εἴποι τις ἂν τὸν δ καὶ ς καὶ θ {δ ς θ 33} μὴ εἶναι ἐλάχιστους ἐν ἡμιολίῳ λόγῳ· οὐδὲ γὰρ εἰσι πυθμένες τῶν ἡμιολίων· ὁ β γὰρ καὶ ὁ γ εἰσι πυθμένες τοῦ ἡμιολίου. πρὸς ὃ λέγομεν ὅτι δύο μὲν ἐφεξῆς ἡμιόλιοι ἔσχατοι καὶ πυθμένες εἰσιν ὁ β καὶ ὁ {ὁ om. 33} γ, τρεῖς δὲ ἐφεξῆς ἐλάττονες τοῦ δ καὶ ς καὶ θ {δ ς θ 33} οὐκ εἰσιν. ὡσαύτως δὲ καὶ δύο μὲν ἐπιτρίτους ἐλάχιστους ἔστιν εὐρεῖν τὸν γ καὶ τὸν δ, τρεῖς δὲ ἐλάσσονας {ἐλάχιστους 67 : -ττονος 1549 2377} τῶν θ καὶ ιβ καὶ ις {θ ιβ ις 33} οὐδαμῶς. καὶ ἐπὶ τῶν ἄλλων ὡσαύτως· εἰκότως οὖν οὗτοι τοὺς ἄκρους τετραγώνους ἔχουσιν.

C.3. Recension III

Tarán's edition.

ρνζ. οἱ γὰρ ἄκροι πάντως τετράγωνοι εὐρεθήσονται, ὡς ἐπὶ μὲν τῶν ἡμιολίων τοῦ δ καὶ τοῦ ς καὶ τοῦ θ, ὁ δ καὶ ὁ θ· ἐπὶ δὲ τῶν ἐπιτρίτων τοῦ θ καὶ τοῦ ιβ καὶ τοῦ ις, ὁ θ καὶ ὁ ις· καὶ ἐπὶ πάντων τῶν εἰδῶν τετράγωνοί εἰσιν οἱ ἄκροι, καὶ τοῦτο εὐλόγως, ἐπειδὴ δέδεικται γραμμικῶς ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ὥσι πρὸς ἀλλήλους τὸν αὐτὸν λόγον ἔχοντες, οἱ ἄκροι αὐτῶν τετράγωνοί εἰσιν. ἐπεὶ οὖν καὶ ἐπὶ τῶν ἡμιολίων ὁ δ καὶ ὁ ς καὶ ὁ θ ἐλάχιστοί εἰσι, διὰ τοῦτο οἱ ἄκροι τετράγωνοί εἰσιν. ἀλλ' ἴσως εἴποι τις ὅτι “οὐκ εἰσιν ἐλάχιστοι, οὐδὲ γὰρ πυθμένες εἰσι τῶν ἡμιολίων· ἰδοὺ γὰρ ὁ γ τοῦ β ἡμιόλιος.” φαμέν ὅτι δύο μὲν ἀριθμοὶ ἡμιόλιοι ἐλάχιστοι εὐρίσκονται, ὡς ὁ β καὶ ὁ γ, τρεῖς δὲ οὐκέτι· οὐκ ἂν γὰρ ὑποκάτω τοῦ δ καὶ τοῦ ς καὶ τοῦ θ τρεῖς ἡμιολίους εὕρῃς. ὡσαύτως δὲ καὶ δύο μὲν ἐπιτρίτους εὐρίσκεις, τὸν γ καὶ τὸν δ· τρεῖς δὲ ὑποκάτω τοῦ θ καὶ τοῦ ιβ καὶ τοῦ ις οὐκ ἂν εὕροις. καὶ ἐπὶ πάντων τῶν ἄλλων ὁμοίως, ὥστε εἰκότως τοὺς ἄκρους τετραγώνους ἔχουσιν.

C.4. Recension IV

My own critical text. B and E are the sigla I have assigned to the two branches of the manuscript tradition, and coincide with extant manuscripts. It turns out that E is a recension of B.³² Segments of text drawn from *Ar.* are in red; variant readings are in blue.

ἐπὶ πάσης τῶν διαζευχθεῖσων ἦτοι ἐπὶ πάσης {ἐπὶ πάσης — ἐπὶ πάσης] ante τῶν sp. 4 litt. B : sp. 14-18 litt. ἦτοι E} τῆς ὀρθῆς καὶ ἀντεστραμμένης λήψεως – διέζευγται γὰρ ἡ κατ' εὐθείαν καὶ κατὰ ἀντιστροφὴν λήψις – ἀφ' ἧς τῆς ὀρθῆς καὶ ἀντεστραμμένης λήψεως ἀμφοτέραι γεννῶνται αἱ σχέσεις. εἰ μὲν κατ' εὐθείαν ληφθῇ, γεννᾶται ἡ πολλαπλασιαστικὸς σχέσις· εἰ δὲ ἀντεστραμμένως, ἡ ἐπιμερής. ὁ μὲν ἔσχατος τετράγωνος, ἦτοι ὁ δ' ἔσχατος δὲ ἐστὶ διότι, εἰ ἀντιστρέφεις καὶ λάβῃς τὸν θ πρῶτον, ἔσχατος καταλιμπάνεται ὁ δ. ἐπὶ τοῦ ἡμιολίου, οἷον εἰ εἴπω ἴσον πρῶτῳ θέσθαι καὶ ἐκθήσω τὸν θ, εἶτα ἴσον πρῶτῳ καὶ δευτέρῳ καὶ ἐκθήσω τὸν {τὸν om. E} ιε, ἔσχατος κατελείφθῃ {-λήφ- B} ὁ δ τετράγωνος. οὗτος οὖν ὁ αὐτὸς μένει, καὶ οὐ γίνεται ὁ δ ἀρχὴ καὶ κορυφὴ ἐλάττονός τινος ἀριθμοῦ ἢ τοῦ ἐπιτρίτου ἢ τοῦ ἐπιτετάρτου, ἀλλ' ὁ αὐτὸς μένει καὶ οὐ μεταβαίνει ἀπὸ τοῦ ἡμιολίου εἰς ἕτερον ἐλάττονα. ὁ δὲ θ τετράγωνος ὢν καὶ αὐτὸς πρῶτος δὲ ἀντεστραμμένως μεταβαίνει εἰς τὸν ἐλάττονα (ἦτοι τὸν ἐπίτριτον)· γίνεται γὰρ κορυφὴ καὶ ἀρχὴ τοῦ ἐπιτρίτου θ ιβ ις, ὁ δὲ ἐπίτριτος ἐλάττων ἐστὶ {ἐλάττον B} τοῦ ἡμιολίου· πᾶν γὰρ μέγεθος εἰς δύο μείζονα τέμνεται τὰ ἡμίση· τέμνεται δὲ εἰς ἐλάττονα, εἰ διαιρεθῇ εἰς τρία μέρη. καὶ πάλιν ὁ θ ἐν τῷ ἐπιτρίτῳ ἔσχατος ὢν ἀντεστραμμένως οὐ μεταβαίνει εἰς ἐλάττονα ἀλλὰ μένει ὁ αὐτός. Ὁ δὲ ις τετράγωνος πρῶτος ὢν κατ' ἀντιστροφὴν – εἰ γὰρ ἀντιστρέψεις καὶ εἴπῃς ἴσον πρῶτῳ θέσθαι καὶ ἐκθήσεις τὸν ις, ἔστιν ὁ ις ἀντεστραμμένως πρῶτος – ὁ δὲ θ ἔσχατος. ὁ οὖν ις μεταβαίνει εἰς {καὶ B} τὸν ἐλάττονα τοῦ ἐπιτετάρτου καὶ ἀρχὴ γίνεται αὐτοῦ. ις κ κε. Ὁ δὲ ἐπιτετάρτος ἐλάττων ἐστὶ τοῦ ἐπιτρίτου καθ' ὃν τρόπον καὶ ὁ ἐπίτριτος τοῦ ἡμιολίου. οὗτος δὲ ὁ λόγος καὶ ἡ μετάβασις τοῦ πρώτου κατ' ἀντιστροφὴν εἰς τὸν ἐλάττονα ἐπὶ μόνων τῶν ἐπιμορίων σώζει. πάντες δὲ οἱ {οἱ om. B} ἄκροι. δ ς θ· καὶ ὁ δ καὶ ὁ θ τετράγωνοι· θ ιβ ις· καὶ ὁ θ καὶ ὁ ις τετράγωνοι· δ ι κε· καὶ ὁ δ καὶ ὁ κε τετράγωνοι, καὶ ἀπλῶς πάντες οἱ ἄκροι τετράγωνοι· δέδεικται γὰρ ἐν γεωμετρίᾳ ὅτι ὅταν τρεῖς ἀριθμοὶ ἴσῃ ἀναλογίαν ἔχωσι πρὸς ἀλλήλους {ἄλληλα B} (οἷον δ ς θ· ὃν γὰρ λόγον ἔχει ὁ ς πρὸς τὸν δ, τοῦτον ἔχει ὁ θ πρὸς τὸν ς), τούτων οἱ ἄκροι εἰσὶ τετράγωνοι.

C.5.1 The Adjunct to the text of the *Introductio arithmetica* in some manuscripts

Same conventions as for *Ar.* I.23.15 above.

A

Ambros. B 77 sup., f. 58r; Ambros. G 62 sup., f. 43r-v; Bucur., BAR gr. 620, pp. 224-225; Gott. philol. 66, f. 141r in Rec. I, f. 141r-v (m. rec. κείμενον); Laur. Conv. Soppr. 30, f. 43r; Laur. Plut.

³² These statements are proved in Acerbi, “La tradition” (cit. n. 27).

28.35, f. 31v; Laur. Plut. 58.29, f. 153r; Marc. gr. Z. 317, f. 23v; Marc. gr. Z. 316, f. 61r-v; **Marc. gr. Z. 318, f. 21r**; Marc. gr. Z. 319, f. 39r; **Marc. gr. Z. 320, f. 52v**; Marc. gr. Z. 330, f. 302r; Marc. gr. Z. 333, f. 61v; Marc. gr. Z. 514, f. 24v; Marc. gr. Z. 595, f. 32v; **Monac. gr. 482, f. 133r**; Mutin. α.U.9.7, f. 38v (marg.); Neapol. III.C.6, f. 32r; Oxon. Bodl. Holkham 71, f. 278v; **Oxon. Bodl. Laud. gr. 44, p. 90**; Par. Coislin 174, f. 102v (marg. Rec. I); Par. gr. 2373, f. 17r; **Par. gr. 2376, f. 78v**; Par. gr. 2762, f. 41r-v; Par. gr. 2480, pp. 343-345, 346 (in Rec. I); Par. gr. 2531, f. 80v (marg.); **Par. suppl. gr. 450, f. 35r**; Roma, Arch. S. Paolo 24C, f. 55r marg. Rec. I; Scorial. Σ.II.15, f. 171v; Scorial. X.I.9, f. 47rv; Vat. gr. 1026, ff. 63v-65r (m. rec.); Vat. gr. 1040, f. 20v; Vat. gr. 1051, f. 20v (m. rec.); Vat. Ottob. gr. 310, f. 142r; Vindob. phil. gr. 62, f. 24r

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοί εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ' ἀλλήλαις γενομένων, καὶ ὁ μὲν πρῶτος τῆς ἀπογεννώσεως εἰς τὸν ἐλάττονα τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ μείζων τετράγωνος ὁ αὐτός ἐστιν.

Ambros. B 77 sup. γενομένης | γεννηθ- | Ambros. G 62 sup.: marg. μέχρι τούτου εὐρίσκεται τὸ πρῶτον βιβλίον τὸ δ' ἄλλο περιττόν | **ἐπαλλήλως** | **om. ὁ (ult.)** | Laur. Conv. Soppr. 30: **om. ὁ αὐτὸς** | **Laur. Plut. 28.35**: μὲν τοῖς | **ἐπαλλήλως** | **om. ὁ (ult.)** | Marc. gr. Z. 316 & Scorial. X.I.9: γεννωμένης μεταβαίνει | Marc. gr. Z. 317: **om. ἐκκειμένων** | **Marc. gr. Z. 318**: **om. ὁ αὐτὸς** | Marc. gr. Z. 319: **λεγομένων** | **Marc. gr. Z. 320**: **ἀλλήλας** | **καὶ {ὁ s.l. m. rec.} μείζων** | **ex ὁ αὐτὸς fecit ὡσαύτως m. rec.** | Marc. gr. Z. 333: ἀπογεννωμένης μεταβαίνει | **Monac. gr. 482**: **ἐπαλλήλως** | γρ. ἐπ' ἀλλήλαις marg. m. rec. | **om. ὁ (ult.)** | Mutin. α.U.9.7: **ἀλλήλ(comp. legi nequit)** | **ἀμφοτέρ(comp. legi nequit)** | Neapol. III.C.6: **om. ἐστιν** | Oxon. Bodl. Holkham 71: μὲν τὴν | -γεν-ubique | **om. ὁ (ult.)** | **Oxon. Bodl. Laud. gr. 44**: **ἐπαλλήλως** | **om. ὁ (ult.)** | **Par. gr. 2376**: **om. ὁ αὐτὸς** | Par. gr. 2762: αὐτοῦ | Par. gr. 2531: **ἀλλήλως** | **ἀμφοτέροις** | **Par. suppl. gr. 450**: **ἐπαλλήλως** | **om. ὁ (ult.)** | Scorial. Σ.II.15: μὲν τὴν | **λεγομένων** | -γεν-ubique | γενομένης | **om. ὁ (ult.)** | Vat. gr. 1026: **om. ὁ (ult.)** | Vindob. phil. gr. 62: ἀλλήλας

B

Ambros. I 8 sup., f. 34r; [Athens, EBE 1115, p. 59]; Bonon BU 2263, f. 16v; Hamburg, S-UB, philol. 88, f. 24r; Kharkow, UL, 269-p, 369-c, f. 87v; Leiden, BRU, Periz. Q° 39, f. 23v; Mutin. α.W.3.1, f. 18r; Oxon. Bodl. Selden Supra 20, f. 17r; Par. gr. 2372, ff. 25v-26r; Par. gr. 2479, f. 92v (m. rec. κείμενον); Par. gr. 2483, p. 153; Scorial. P.II.3, f. 436r; Vat. gr. 195, f. 43r; Vat. gr. 256, f. 316v; Vat. gr. 2387, f. 48v

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων **πάντες** οἱ ἄκροι τετράγωνοί εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ' ἀλλήλαις γενομένων, καὶ ὁ μὲν πρῶτος τῆς ἀπογεννώσεως εἰς τὸν ἐλάττονα τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ μείζων τετράγωνος ὁ αὐτός ἐστιν.

Athens, EBE 1115: deficit usque ad μεταβαίνει | ἀμφοτέρες (male legit comp. Vat. gr. 256) | Leiden, BRU, Periz. Q° 39: om. ἐν | Par. gr. 2372: ἀπογεννώσεις | Par. gr. 2479: om. τῆς (pr.) | Scorial. P.II.3: ἀπὸ γεννέσης | γενομένης

C

Vat. gr. 196, f. 15r

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων {om. ἐκθέσεων} πάντως οἱ ἄκροι τετράγωνοί εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ' ἀλλήλαις γενομένων, καὶ ὁ μὲν πρῶτος τοῦ ἀπὸ γεννώσεως εἰς τὸν ἐλάττονα τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ μείζων τετράγωνος ὁ αὐτός ἐστιν.

ex ἐκκειμένων fecit ἐκθέσεων m. rec.

D

Hamburg, S-UB, philol. 89, p. 368 (marg.); Par. gr. 2374, f. 21v; Scorial. Υ.III.12, f. 21r; Vat. gr. 198, f. 19r (marg. keyed to the first word of Ar. I.23.15)

ἐπὶ πασῶν {om. μέντοι} τῶν ἐκκειμένων σχέσεων πάντως μὲν οἱ ἄκροι τετράγωνοί εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ' ἀλλήλαις μηκυνθισῶν, καὶ ὁ μὲν πρῶτος τῆς ἀπογεννώσεως εἰς τὸν ἐλάττονα τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ ὁ μείζων τετράγωνος ὁ αὐτός ἐστιν.

Par. gr. 2374: ἐκθέσεων | πάντες | Scorial. Υ.III.12: [[ἐλήφθησαν]] ante ἐπ' ἀλλήλαις | des. μηκυνομένων

E

Berol. Phillipps 1549, f. 84v; Bucur., BAR gr. 520, p. 118; Erlangen, UL A.8, ff. 38v-39r; Cizensis 67, f. 51v; Monac. gr. 76, f. 248r; Oxon. Lincoln gr. 33, f. 110r; Par. gr. 2377, f. 84v; Vat. gr. 1411, f. 76v

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων θέσεων πάντως οἱ ἄκροι τετράγωνοί εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ' ἀλλήλας γινομένων, καὶ ὁ μὲν πρῶτος τῆς ἀπογεννώσεως εἰς τὸν ἐλάττονα τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ μείζων τετράγωνος ὡσαύτως ἐστιν.

Bucur., BAR gr. 520: γεννηθήσας | Cizensis 67: ἐλάχιστον | Erlangen, UL A.8: γενομένης | Par. gr. 2377: γενομένης

F

Ambros. P 121 sup., f. 130v; Guelf. 36 Gud. gr., f. 17v; Leiden, BRU, BPG 74G, f. 25r; Marc. gr. Z. 309, f. 193v; Marc. gr. Z. 592, f. 37r; Neapol. III.C.1, f. 20v; Norimb. Cent. V app. 36, f. 18v; Par. Coislin 174, f. 58v; Par. gr. 2107, f. 85r; Par. gr. 2450, f. 111v; Vat. Barb. gr. 273, f. 122v; Vindob. phil. gr. 220, f. 62r

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοι εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ' ἀλλήλας γενομένων, καὶ ὁ μὲν πρῶτος τῆς ἀπογεννώσεως εἰς τὸν ἐλάττονα τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ μείζων **τετράγωνοι ὡσαύτως εἰσίν.**

Ambros. P 121 sup.: καὶ ὁ μείζων | τετράγωνος ὡσαύτως {ὁ αὐτὸς s.l. m. rec.} ἐστίν | Leiden, BRU, BPG 74G: τετράγωνος ὡσαύτως ἐστίν | Marc. gr. Z. 592: τῶν παρ' αὐτῶν | Neapol. III.C.1: –γεν–ubique | καὶ [[οὐ]] μείζων | τετράγωνος ὡσαύτως ἐστίν | Norimb. Cent. V app. 36: om. πάντως | καὶ ὁ μείζων | Par. Coislin 174: om. καὶ (sec.) | τετράγωνος ὡσαύτως ἐστίν | Par. gr. 2107: **πλ(ευ)ρ(ῶν)** | **totum locum fenestrat** | Par. gr. 2450: τετράγωνος ὡσαύτως ἐστί | Vat. Barb. gr. 273: **τετράγωνος ὡσαύτως ἐστί** | Vindob. phil. gr. 220: καὶ οὐ μείζων | τετράγωνος ὡσαύτως ἐστίν

Glosses to the Adjunct

Guelf. 36 Gud. gr.

{marg. μέχρι τούτου εὐρίσκεται τὸ πρῶτον βιβλίον τὸ δ' ἄλλο περιττόν} ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοι εἰσιν, οἱ δὲ μέσοι ἐκ τῶν **πλ(ευ)ρ(ῶν)** {marg. πλευρῶν} αὐτῶν ἐπ' ἀλλήλας {s.l. –ων} γενομένων, καὶ ὁ μὲν πρῶτος τῆς ἀπογεννώσεως εἰς τὸν ἐλάττονα τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ μείζων τετράγωνοι {s.l. –ς} ὡσαύτως εἰσίν {marg. αὐτὸς ἐστίν}.

C.5.2 The scholium to the Adjunct in Recension II manuscripts

Berol. Phillipps 1549, f. 88r-v; Cizensis 67, f. 36r; Monac. gr. 76, f. 160r-v; Oxon. Lincoln gr. 33, f. 110v; Par. gr. 2377, f. 88r-v; Vat. gr. 1411, f. 60r

<ρ>νθ {om. 76}. <πάντως οἱ ἄκροι.> τοὺς μὲν ἄκρους πασῶν τῶν τοιούτων σχέσεων τετραγώνους {–ος sed –ους s.l. 1549} ἔφησεν εἶναι, καὶ ἡμεῖς εἵπομεν δεῖκνυσθαι τοῦτο ἀναγκαίως καὶ παρὰ τῷ γεωμέτρῃ. τοὺς δὲ μέσους φησὶν ἐκ τῶν πλευρῶν τῶν ἄκρων τετραγώνων γίνεσθαι, τουτέστι πολλαπλασιαζομένης {πολυ– 33} τῆς {τῆς om. 1549 2377} τοῦ ἐνὸς πλευρᾶς ἐπὶ τὴν τοῦ ἐτέρου. δεῖκνυσι γὰρ καὶ περὶ τούτου ὁ {ὁ om. 76} γεωμέτρης ὅτι δύο μεταξὺ τετραγώνων ἀριθμῶν {ἀ. τ. 33} εἰς ἀνάλογον {–ος 67} ἐμπίπτει, καὶ οὗτος ὑπὸ τῶν πλευρῶν τῶν τετραγώνων πολλαπλασιαζομένων ἐπ' ἀλλήλας γίνεται.

C.6. The scholia in Matrit. 4678, f. 25v; cf. Kharkow, UL, 269-p, 369-c, f. 87v [a], Laur. Conv. Soppr. 30, ff. 42v-43r [a], Marc. gr. Z. 309, f. 194r [ab], Oxon. Bodl. Holkham 71, f. 278r [a], Par. gr. 2483, p. 149 [a], Scorial. P.II.3, f. 432r-v [a], Vat. gr. 1026, ff. 63v-64r [ab]

a. ἐφ' ὧν ἐξέθετο σχέσεων ἐν τοῖς ὑποδείγμασιν, ἐὰν τάξωμεν πρῶτον τὴν ἡμιολίαν, εἴτα τὴν ἐπίτριτον καὶ ἐξῆς τὸν ἐπιτέταρτον, εὐρήσομεν {–σαιμ– 30} ὅτι ὁ μὲν τῆς ἡμιολίας

πρῶτος τετράγωνος ἐφ' ἑαυτοῦ μένει, ὁ δὲ ἔσχατος τετράγωνος μεταβαίνων εἰς τὸν ἐπι-
 τρίτον ἐλάττονα ὄντα τοῦ ἡμιολίου πρῶτος αὐτοῦ γίνεται. καὶ πάλιν αὐτὸς μὲν ὁ πρῶτος
 τοῦ ἐπιτρίτου οὐ μετέρχεται εἰς τὸν ἐπιτέταρτον, {usque huc legi nequit 71} ὁ δὲ ἔσχα-
 τος τοῦ ἐπιτρίτου μετερχόμενος πρῶτος γίνεται τοῦ ἐπιτετάρτου ἐλάττονος {-ον 4678}
 ὄντος τοῦ ἐπιτρίτου.

Kharkow, UL, 269-p: lemma ἐπὶ πασῶν δὲ τῶν διαzeugθειςῶν | Par. gr. 2483: lemma ἐπὶ πα-
 σῶν δὲ τῶν διαzeugθειςῶν | marg. σημείωσον ὅτι ξένον | marg. οὐκ ἔστι τοῦ Φιλοπόνου m.
 rec. | Kharkow, UL, 269-p & Laur. Conv. Soppr. 30 & Oxon. Bodl. Holkham 71 & Par. gr. 2483
 & Scorial. P.II.3: ἡμιόλιος masc. ubique | om. τὴν (sec.) | ἐφ' ἑαυτὸν (ἑαυτοῦ 30) | ἐλάττω | om.
 μετερχόμενος | Marc. gr. Z. 309: ἐλάττω | om. ὁ πρῶτος | Vat. gr. 1026: om. ὁ (ter.)

b. <πάντως δὲ οἱ ἄκροι τετράγωνοι.> οὐ πάντες τῶν ἐχόντων ὁμοίως τὰς σχέσεις, ἀλλὰ
 τῶν ἐκκειμένων ἐνταῦθα. τὸ μέντοι “τοὺς πρώτους μὴ μεταβαίνειν τοὺς δὲ ἐσχάτους
 μεταβαίνειν εἰς τοὺς ἐλάττονας” καὶ ἐπὶ τῶν μὴ ἐχόντων τοὺς <ἄκρους> τετραγώνους
 συμβαίνει. ἐξεθέμεθα σαφηνείας χάριν ὑποδείγματα ἐπὶ διαγραμμάτων καὶ τῶν ἐχόντων
 τοὺς ἄκρους τετραγώνους καὶ τῶν μὴ ἐχόντων. οἷον τῶν μὲν ἐχόντων ταῦτα: *** τῶν δὲ
 μὴ ἐχόντων πάλιν ταῦτα: η, ιβ, ιη. ιη, κδ, λβ. λβ, [ν]{μ}, ν. <ν>, [οβ]{ξ}, οβ.

Marc. gr. Z. 309: πάντων εἰσὶ τῶν | ἐκτιθέμεθα δὲ | ἔτι διαγραμμάτων | τάδε δ ς θ, θ ιβ ις, *ις* κ
 κε | λβ μ ν, ν ξ οβ | Vat. gr. 1026: πάντων δὲ τῶν | ἐξετιθέμεθα | τάδε dein des.

Glosses to the Adjunct

Marc. gr. Z. 309, f. 193r

ἐπὶ πασῶν δὲ τῶν διαzeugθειςῶν {s.l. ἡγουν τῆς μονάδος καὶ δυάδος} καὶ ἀφ' ἧς
 ἀμφοτέραι, ὁ μὲν ἔσχατος τετράγωνος {s.l. ὁ δ} ὁ αὐτὸς μένει, ὁ δὲ πρῶτος {s.l. τετρά-
 γωνος πρῶτος ἢ μονάς} εἰς τὸν ἐλάττονα {s.l. τὸν θ· διότι τοῦ ς οὔτος ὁ θ ἡμιόλιος}
 μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

Vat. gr. 1026, f. 63v

ἐπὶ πασῶν δὲ τῶν διαzeugθειςῶν {s.l. τὴν μονάδα καὶ δυάδα καὶ τριάδα καὶ τετράδα τὰς
 ἐκ τῶν γ' μονάδων γεννηθεῖς[ας ...] τῆς ἀντιστροφῆς αὐτῶν [...]} καὶ ἀφ' ἧς ἀμφοτέραι,
 ὁ μὲν ἔσχατος τετράγωνος {s.l. οἷον ὁ δ} ὁ αὐτὸς μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα
 {s.l. τὸν θ· οὔτος γὰρ ἡμιόλιος, ὁ δ' ἡμιόλιος λόγος ἐλάττων} μεταβαίνει, πάντως δὲ οἱ
 ἄκροι τετράγωνοι.

C.7. Glosses and scholia in Monac. gr. 482, f. 132v-133r, and Laur. Plut. 28.35, f. 31r-v [only the glosses]

Signs anchor the scholia on the underlined syntagms. The quotation of *Ar.* I.23.15 is in red. Cf. Texts **C.8a** and **C.9a** for the readings in blue.

Ar. I.23.15

ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν {s.l. ἀναλογιῶν} καὶ ἀφ' ἧς {s.l. ἀναλογίας} ἀμφοτέραι, ὁ μὲν ἔσχατος {s.l. ὁ ἐλάττων ἡγουν οἱ δύο ἄκροι} τετράγωνος ὁ αὐτὸς μένει, ὁ δὲ πρῶτος {s.l. ἡγουν μείζων} εἰς τὸν ἐλάττονα μεταβαίνει, πάντως {s.l. ἡγουν ἐξ ἀνάγκης} δὲ οἱ ἄκροι τετράγωνοι {s.l. signum quadratum}.

a. διαζευχθείσας λέγει τὴν μονάδα καὶ δυάδα καὶ τετράδα τὰς ἐκ τῶν τριῶν μονάδων γεννηθείσας κατὰ τὸ πρόσταγμα τὸ λέγον “πρῶτον πρῶτῳ ἴσον ποιήσον”, καὶ πάλιν τὰς γεννηθείσας ἀπ' αὐτῶν τῆς τε μονάδος καὶ τριάδος {lege δυάδος} καὶ τετράδος, αἵτινές εἰσι μία, τρεῖς, θ, καὶ τὰς ἐκ τῆς ἀντιστροφῆς αὐτῶν τῶν δ, β, α, αἵτινές εἰσι δ, ζ, θ. ἐπὶ πασῶν οὖν τούτων τῶν διαζεύξεων καὶ ἀφ' ἧς πρώτης διαζεύξεως, τουτέστι τῶν τριῶν μονάδων, ὁ μὲν ἔσχατος τετράγωνος (οἷον ὁ δ) ὁ αὐτὸς μένει, ὁ δὲ πρῶτος (ἥτοι ἡ μονάς· καὶ γὰρ αὕτη πάντα δυνάμει) εἰς τὸν ἐλάττονα μεταβαίνει, ἥτοι τὸν θ· ὁ γὰρ θ ἡμιόλιος, ὁ δὲ ἡμιόλιος λόγος ἐλάττων τοῦ διπλασίου, ὥσπερ καὶ ὁ ἐπίτριτος τοῦ ἐλάττονος.

b. ἐδείχθη παρὰ τῷ γεωμέτρῳ ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ὥσι πρὸς ἀλλήλους τὸν αὐτὸν λόγον ἔχοντες, οἱ ἄκροι αὐτῶν τετράγωνοι εἰσι.

Adjunct

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοι εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν {s.l. τῶν τετραγώνων} ἐπ' ἀλλήλως γενομένων, καὶ ὁ μὲν πρῶτος {s.l. μείζων ὅρος} τῆς ἀπογεννώσης {s.l. ἐκθέσεως} εἰς τὸν ἐλάττονα τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος {s.l. ἐλάττων} καὶ μείζων τετράγωνος αὐτός ἐστιν.

C.8. Four scholia in Ambros. G 62 sup., ff. 65v-66v [abcd], Marc. gr. Z. 309, ff. 193v-194r [ab], Marc. gr. Z. 316, f. 61r-v [c], Marc. gr. Z. 318, ff. 20v-21r [abc], Marc. gr. Z. 319, ff. 39r-40r [bc], Marc. gr. Z. 333, f. 61r [a{part.}c], Mutin. α.T.8.14, f. 27v [m. rec.; a{part.}c], Scorial. X.I.9, ff. 46v-47r [c]

In items **a.** and **d.**, the quotation of Ar. I.23.15 is in red. Cf. Texts **C.7a** and **C.9a** for the first reading in blue in item **a.** In item **d.**, integrations in lacuna, some of which *exempli gratia*, are enclosed in braces. Because of its length, I have introduced paragraphs in item **d.**

a. tit. {τοῦ} φιλοσόφου Ἐφεσίου σχόλιον εἰς τὸ ῥητὸν τὸ *** 62 : τοῦ φιλοσόφου Ἐφεσίου σχόλιον 318

διαζευχθείσας λέγει τὴν μονάδα καὶ δυάδα καὶ τετράδα τὰς ἐκ τῶν τριῶν μονάδων γεννηθείσας κατὰ τὸ πρόσταγμα τὸ λέγον “πρῶτον πρῶτῳ ἴσον ποιήσον”, καὶ πάλιν τὰς γεννηθείσας ἀπ' αὐτῆς τῆς μονάδος καὶ δυάδος καὶ τετράδος, αἵτινές εἰσιν α γ θ, καὶ τὰς ἐξ ἀναστροφῆς αὐτῶν τῆς δ β α, αἵτινές εἰσι δ ζ θ {καὶ τὰς — δ ζ θ om. 62 318}. ἐπὶ πασῶν

οὖν τούτων τῶν διαζεύξεων καὶ ἀφ' ἧς πρώτης διαζεύξεως, τουτέστι τῶν τριῶν μονάδων, ὁ μὲν ἔσχατος τετράγωνος (οἷον ὁ δ) ὁ αὐτὸς μένει, ὁ δὲ πρῶτος (ἦτοι ἡ μονάς· καὶ γὰρ αὕτη πάντα δυνάμει) εἰς τὸν ἐλάττωνα μεταβαίνει, ἦτοι τὸν θ· ὁ γὰρ θ ἡμιόλιος, ὁ δ' ἡμιόλιος λόγος ἐλάττων τοῦ διπλασίου, ὥσπερ καὶ ὁ {ὁ om. 318} ἐπίτριτος τοῦ ἡμιολίου.

Marc. gr. Z. 309: om. tit. | ποιῆσαι | ἀπ' αὐτῶν τῆς τε μονάδος | Marc. gr. Z. 333: om. tit. | καὶ τὴν δυ. καὶ τὴν τε. | des. ποιῆσαι (sic) dein inc. c | Mutin. α.T.8.14: tit. εἰς τὸ ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν ἐξήγησις Εὐστρατίου Νικαίας | καὶ τὴν δυ. καὶ τὴν τε. | des. ποιήσον dein inc. c

b. tit. εἰς τὸ αὐτὸ κρείττων ἐξήγησις 309 : εἰς τὸ αὐτὸ τοῦ Δυρραχίου 318 : σχόλιον τοῦ Δυρραχίου 319

ἔφησεν ἀνωτέρω ὅτι ληφθήσονται {-σεται 309 319} ἡμῖν “ἅπαντα τὰ τῆς ἀνισότητος εἶδη” ἐκ τινων προσταγμάτων. ἐπάγει τοίνυν παρακολούθημά τι {-ματα 309} τῶν διαζευγνυμένων καὶ ἀπογεννωμένων ὑπὸ {ἀπὸ 309 319} τινος σχέσεως ὀρθῶς {ὡς ὀρθῆς 319} κειμένης ἢ ἀντιστρόφως, καὶ φησιν ὅτι “ἐπὶ πασῶν τῶν διαζευχθεισῶν καὶ ἀφ' ἧς” σχέσεως ἀπετελέσθησαν αἱ διαζευχθεῖσαι ὁ μὲν ἔσχατος ἀριθμὸς τῶν ἀπογεννωμένων ἐκ τῆς πρώτης {ἐκείνης add. 309 319} σχέσεως τετράγωνος καὶ {τε καὶ ὁ 319} αὐτὸς εὑρεθήσεται, ὁ δὲ πρῶτος ἀριθμὸς ἀφ' ἧς {σχέσεως add. 309} αἱ διαζευχθεῖσαι ἐλάττων εὑρίσκεται ἐν τῇ ἐξ αὐτῆς {-οὐ 309} διαζευχθείσῃ. καὶ προκείσθω ἡμῖν σχέσις ἐφήμις {ἡ διπλασιεφημίς 309 : ἡμιόλιος m. rec. 318 : ἡ ἡμιόλιος 319} θ ς δ. ποιήσον οὖν πρῶτον πρῶτῳ ἴσον, δεύτερον πρῶτῳ ἴσον καὶ δευτέρῳ ἅμα {πρῶτῳ ἅμα καὶ δευτέρῳ 309 : πρῶτῳ ἅμα καὶ δευτέρῳ ἴσον 319}, καὶ τρίτον πρῶτῳ ἅμα καὶ δυσὶ δευτέροις {ἅμα add. 319} καὶ τρίτῳ {καὶ τρίτῳ καὶ δις τῷ δευτέρῳ 309}· γίνονται οὖν θ ιε κε. ἀντίστρεψον {στρέψον 319} οὖν {οὖν om. 309} αὐθις τὴν πρώτην ἐκείνην σχέσιν {ἐ. τ. π. σ. 309}· καὶ γενήσονται πάλιν δ ι κε. ὅρα οὖν ὅτι τῶν μὲν {μὲν om. 319} διαζευχθεισῶν καὶ ἀπογεννηθεισῶν {-εἰς 309} ἀπὸ τῆς πρώτης ἐκείνης σχέσεως οἱ ἄκροι καὶ ἔσχατοι τετράγωνοί τε εἰσι καὶ οἱ {τέ εἰσι καὶ οἱ om. 309} αὐτοί, ὁ δὲ πρῶτος ἀριθμὸς {ἀριθμὸς om. 309} τῆς ἀπογεννώσεως εἰς τὸν ἐλάττωνα {ἐλάττω 309} μεταβαίνει, κἂν τε ἐξ εὐθείας κἂν τε ἀντιστρόφως ἀπογεννήσῃ {-ης 62 318 : γενν- 319}, οἷον ὁ θ ς δ γεννήσας τὸν θ τὸν ιε καὶ τὸν κε ἐλάττων ἐστὶν αὐτῶν {α. ἐ. 319}. ὁμοίως δὲ {δὲ om. 309} κἂν ἀντιστραφεῖν, ἐλάσσων εὑρεθήσεται καὶ {καὶ om. 309} ὁ πρῶτος αὐτῶν· εἰ γὰρ θήσομεν δ ς θ καὶ ποιήσομεν κατὰ τὰ προστάγματα, εὑρεθήσεται καὶ αὐθις ὁ πρῶτος τῆς ἀπογεννώσεως {τῶν ἀπογεννόντων 319} ἐλάττων τῶν ἀπογεννωμένων.

c. tit. εἰς τὸ αὐτὸ ἐξήγησις Εὐστρατίου τοῦ Νικαίας 62 : Εὐστρατίου τοῦ Νικαίας 316 X.I.9 : εἰς τὸ αὐτὸ Εὐστρατίου τοῦ Νικαίας ἐξήγησις 318 : εἰς τὸ αὐτὸ τοῦ Νικαίας Εὐστρατίου 319

ἀφ' ἧς ἀμφοτέραι λέγοιτο ἂν ἡ ἐξ ἑαυτῆς {αὐτῆς 316 : αὐτῶν 319} ἔχουσα δύο ἀποτελουμένας τῷ ὀρθῶς τε {τε om. 333 α.T.8.14} κεῖσθαι καὶ {τῷ add. 316 319 X.I.9} ἀναστρέφεισθαι, διεζευγμέναι δὲ αἱ γεννώμεναι ἐξ αὐτῆς· οἷον ἡ α β δ διπλασία οὖσα σχέσις καὶ ἐκ

πρώτης γεγонуῖα τῆς {τῆς om. 316 319 X.I.9} ισότητος, ὀρθῶς μὲν κειμένη τὴν α {τῶν πρῶτον 319} γ θ τριπλασίονα {-σίαν 316 319 X.I.9} σχέσιν ἀπεκύησεν ἀναστραφεῖσα δὲ τὴν δ ς θ ἡμιολίαν. ἐκείνη μὲν οὖν ἀφ' ἧς ἀμφοτέραι, αὗται δὲ {δὲ om. 319} αἱ διαζευχθεῖσαι. “διαζευχθεῖσαι” δὲ {ὀνομά lac. 62} ζονται ὡς {ἐξ add. 316 319 X.I.9} ἐναντίων γινόμεναι θέσεων· ἐναντίον {-α 333 α.T.8.14} γὰρ ὀρθότης τε καὶ ἀντιστροφή {ἀνα- 316 319 X.I.9}, καὶ τὰ ἐναντία διέστηκε {διέστραπτραι 333 α.T.8.14} καὶ διέζευκται· ἀρκεῖ γὰρ τοσοῦτον ἐπὶ τῷ παρόντι {ἀρκεῖ — παρόντι om. 333 α.T.8.14 316 X.I.9}. συμβαίνει οὖν τῇ {τῆς X.I.9} ἀφ' ἧς ἀμφοτέραι {-τερα 319}, ὁπόταν {-τε 319} γεννώσα ληφ{θη, τὸν lac. 62} μὲν ἔσχατον ὅρον ἐν τῷ αὐτῷ λόγῳ τηρούμενον {τυρ- X.I.9} ἔχειν, τὸν δὲ πρῶτον εἰς τὸν μετ' αὐτὸν ἀποβαίνοντα, εἴτε ὁ ἐλάττων πρῶτος ἐστὶν ὀρθῶς κειμένης αὐτῆς, εἴτε ὁ μείζων πρῶτος τῶν ὅρων ὁ δ' ἐλάττων ἔσχατος ἀναστρεφομένης τῆς σχέσεως. τὸ δὲ αὐτὸ τοῦτο καὶ {καὶ om. 316 319 X.I.9} ταῖς διεzeugμέναις συμβαίνει, ὁπόταν ὡς γεννώσαι ληφθεῖεν ἐτέρας {-αις 319}. “εἰς τὸν ἐλάττονα” οὖν φησι “μεταβαίνειν τὸν πρῶτον ὅρον” τὸν ὑστερογενέστερον ὅρον {λόγον 316 319 X.I.9} ὀνομάζων “ἐλάττονα”, οἷον τὸν ἡμιόλιον τὸν τριπλάσιον· οὗτοι γὰρ ὑστερογενέστεροι τοῦ διπλασίου ἀπεδείχθησαν. ἴσως δὲ {δ' ἂν 316 319 X.I.9} βιαίως {βίαιον 333 α.T.8.14} δόξει {-ῇ 316 319 X.I.9} “ἐλαττον” καλεῖσθαι τὸ ὕστερον, καίτοι “μείζων” ἂν μᾶλλον δικαίως ὀνομασθήσεται {ὁ. δ. 316 319 X.I.9} ὁ τριπλάσιος τοῦ διπλασίου. ἀλλὰ σκόπει πῶς λέγομεν· ὁ πρῶτος τῶν ὅρων τῇ ὑστερογενεστέρᾳ οἰκειούμενος σχέσει ὑπόλογος ἐν αὐτῇ {αὐτῷ 62 318 319 333 α.T.8.14} γίνεται, εἴθ' ὁ μείζων εἴθ' {εἴτε bis 333 α.T.8.14} ὁ ἐλάττων εἴη. ἢ οὖν ἐκ προλόγου {-λήψεως 319} ὑπόλογος γίνεται, εἰ ὁ μείζων ἐν τῇ προτέρᾳ εἴη καὶ οὕτως εἰς τὸ ἐλαττον μεταβέβηκεν, ἢ καὶ ὁ ἐλάττων τῆς προτέρας ὑπάρχων σχέσεως ὑπόλογος μὲν καὶ ἐν ἐκείνῃ {-ω 333 α.T.8.14 : -οις X.I.9} ἐστί, μείζονος δὲ ἀριθμοῦ ἐν τῇ μετ' ἐκείνην {-ων X.I.9} σχέσει γεγεννημένος ὑπόλογος, καὶ οὕτως ἔχει {ἴσχει 316 319 X.I.9} τὸ ἐλαττον· ὃ τε γὰρ δ {τέταρτος X.I.9} – πρόλογος ὢν ἐν τῇ ἀντεστραμμένη διπλασίᾳ {-ῶ -ω -ω 316 319 X.I.9}, ἅτε τοῦ δύο ὑπερέχων διπλασίων {lege δύο} {ὕ. διπλάσιον 333 α.T.8.14 : ὑπάρχων διπλάσιως 316 319 X.I.9} – ἐν τῇ {τῷ 316 319 X.I.9} ἡμιολίῳ ὑπόλογος γίνεται ὑψημιόλιος τοῦ ς ἀποκαθιστάμενος, ἢ τε μονὰς – ὑπόλογος οὖσα ὑποδιπλασία – ὑποτριπλασία {-ος -ος 316 X.I.9 : -πᾶ -πᾶ 319} ἐν τῇ {τῷ 316 319 X.I.9} μετ' αὐτὸν γίνεται, ὥσθ' ἐκατέρωθεν συνῆκται τὸ ἐλαττον.

Marc. gr. Z. 333 & Mutin. α.T.8.14: a dein tit. ἄλλως ἢ ἀρχὴ | τὸν [[ἡμιόλιον α.T.8.14]] τριπλάσιον τὸν ἡμιόλιον· οὗτοι γὰρ (ὁ ἡμιόλιος λέγω καὶ ὁ τριπλάσιος) ὑστερογενέστεροι

d. εἰς τὸ αὐτὸ Νικολάου κριτοῦ τοῦ Δισυπάτου

εἰπὼν ὁ Νικόμαχος “ἀπὸ τῆς ισότητος πρώτης οἷον μητρός τινος καὶ ρίζης γεννᾶσθαι τὰ ποικίλα εἶδη τῆς ἀνισότητος” καὶ παραδοὺς κανόνα τούτου ποιητικὸν τὸν λέγοντα “πρῶτον πρῶτῳ ἴσον ποιήσον, δευτέρον πρῶτῳ ἅμα καὶ δευτέρῳ, τρίτον ἴσον πρῶτῳ καὶ τρίτῳ καὶ δυσὶ δευτέροις”, ἐξέθετο παραδειγματικῶς τρεῖς μονάδας, καὶ κατὰ τὸν

παραδοθέντα κανόνα ἔδειξεν ἐξ αὐτῶν γεννᾶσθαι πρῶτον τὸν α β δ διπλασίονα λόγον· εἶτα ἐκ τούτου ὀρθῶς μὲν κειμένου – ὥσπερ καὶ ἀνεφύη – τὸν α γ θ τριπλασίονα, ἀνεστραμμένως δὲ ληφθέντος (ἡγουν δ β α) ἀποτελεῖσθαι τὸν δ ς θ ἡμιόλιον, οὓς λόγους καὶ ἅς σχέσεις διεzeugμένους καλεῖ, κατὰ τὸ ἐμοὶ δοκοῦν διὰ τὸ ἑτερογενές· ἡ μὲν γὰρ τριπλάσιος ὑπὸ τὴν πολυπλασίαν ἀνάγεται ἡ δὲ ἡμιόλιος ἐπὶ τὴν ἐπιμόριον, τὰ δὲ ἑτερογενῇ διέζευκται πάντα καὶ ἀπηλλοτριῶνται ἀλλήλων ἑκατέραν {σχέσιν}· τούτων δὴ τῶν διεzeugμένων ὀρθουμένως τε καὶ ἀναστρεφομένως ἐδίδαξε γίνεσθαι γεννητικὴν ἑτέρων διεzeugμένων σχ{έσεων, οἷον τὴν μὲν} α γ θ τριπλασίαν ἐν μὲν τῷ ὀρθῶς κείσθαι – ὥσπερ καὶ ἀνεφύη – εἶναι γεννητικὴν τῆς α δ ις τετραπλασίας, {ἐν δὲ τῷ ἀνεστραμμένως} κείσθαι θ γ α ἀπογεννᾶν τὴν θ ιβ ις ἐπίτριτον σχέσιν· τὴν δὲ δ ς θ ἡμιολίαν ἐν μὲν τῇ κατ' ὀρθὸν θέσει ἀπογεννᾶν τὴν δ ι κε διπλασιεφήμισυν ἐν δὲ τῇ ἀνεστραμμένῃ τῇ θ ς δ ἀποτελεῖν τὴν {θ ιε κε ἐπιδιμερῇ ἢ} ἐπιδίτριτον σχέσιν.

ταύτη οὖν τῇ μεθόδῳ χρησάμενος ὁ Νικόμαχος εἰς παράστασιν τῆς προχ{ωρήσεως **} τῆς ἀνισότητος καὶ δείξας παραδειγματικῶς – ἐπὶ τῆς ἐν ἡμιολίῳ λόγῳ τῷ δ ς θ σχέσεως τῷ γεννω{μένῳ ἀπὸ τοῦ} διπλασίου τοῦ δ β α – συνίστασθαι ἐκ ταύτης ἀπὸ μὲν τοῦ ἐλάττονος ὅρου τοῦ δ – ὅτε καὶ ὀρθῶς {κεῖται} – τὸν {δ ι κ}ε διπλασιεφήμισυν, ἐκ δὲ τοῦ μείζονος ὅρου τοῦ θ – ὅτε καὶ ἀνεστραπτὰ – γεννᾶσθαι τὸν θ ιε κε ἐπιδιμερῇ ἢ ἐπιδίτριτον, τὸ αὐτὸ δὲ γίνεσθαι καὶ ἐπὶ τῆς ἐν ἐπιτρίτῳ καὶ ἐπιτετάρτῳ λόγῳ σχέσεως, ἐπάγει καθολικῶς ὅτι **ἐπὶ πασῶν τῶν διαzeugθειςῶν** [ὅτε δηλονότι καὶ αὗται ὡς γεννῶσαι ληφθεῖεν ἑτέρας ὀρθούμενα marg. sup. m. rec.] (ὅτε δηλονότι καὶ αὗται ὡς γεννῶσαι ληφθεῖεν ἑτέρας ὀρθούμεναι ἢ ἀναστρεφόμεναι) καὶ ἐπὶ τῆς σχέσεως ἐκείνης **ἀφ' ἧς ὀρθῶς ἢ ἀνεστραμμένως ἐχούσης τίκτονται αὗται αἱ διεzeugμέναι, ὁ μὲν ἔσχατος (τετράγωνος)** (ἡγουν ὁ ἐλάσσων ὅρος) **ὁ αὐτὸς μένει** καὶ ἐν ταῖς ἀπογεννωμέναις αὐτῶν σχέσεσιν (ἡγουν ἐλάττων καὶ ὑπόλογος), πότε δὲ ἐν τῷ ὀρθῶς δηλονότι κείσθαι ἐκάστην αὐτῶν· ἐν μέντοι τῷ ἀνεστράφθαι ταύτας **ὁ πρῶτος** ὅρος (ἡγουν ὁ μείζων) **εἰς τὸν ἐλάττονα μεταβαίνει** ἐν ταῖς ἐξ αὐτῶν ἀπογεννωμέναις σχέσεσι, καὶ ἀντὶ προλόγου γίνεται ὑπόλογος, κἂν ἀμετάβλητος ἢ καὶ ὁ αὐτὸς καθὸ τετράγωνος.

οἷον ἔστω ὡς ἐν παραδείγματι ἡ ἀπὸ τῆς πρώτης ισότητος τῶν τριῶν μονάδων ὡς ἀπὸ μητρὸς τινος καὶ ρίζης ἀποτελεσθεῖσα α β δ σχέσις διπλασία, κατὰ τὸν παρα{δο}θέντα παρὰ τοῦ Νικομάχου κανόνα τὸν λέγοντα “πρῶτον πρῶτῳ ἴσον ποιήσον, δεύτερον πρῶτῳ καὶ δευτέρῳ ἅμα, καὶ τρίτον ἴσον πρῶτῳ καὶ τρίτῳ καὶ δυσὶ δευτέροις”. ὥσπερ οὖν ἐπὶ ταύτης ὀρθῶς μὲν κειμένης – οὕτως ὡς ἀνεφύη – ὁ ἔσχατος τετράγωνος αὐτῆς (ἡγουν ἡ μονάς· αὕτη γὰρ πάντα δυνάμει) ὁ αὐτὸς μένει (ἡγουν ἐλάσσων καὶ ὑπόλογος καὶ ἐν τῇ ἐξ αὐτῆς {ἀποτ}ελεσθείσῃ κατὰ τριπλάσιον λόγον σχέσει τῇ α γ θ) ἀνεστραμμένως δὲ ληφθείσης τῆς αὐτῆς διπλασίας, τουτέστι δ β α, ὁ πρῶτος ταύτης (ἡγουν ὁ μείζων) ὁ δ, κἂν ὁ αὐτὸς μένη καθὸ τετράγωνος, ἀλλ' αὖ εἰς τὸν ἐλάσσονα μεταβαίνει ὅρον καὶ ἀντὶ προλόγου γίνεται ὑπόλογος ἐν τῇ ἐξ αὐτῆς ἀποτεχθείσῃ ἡμιολίῳ σχέσει τῇ δ ς θ.

τὸν αὐτὸν δὴ τρόπον καὶ ἐφ' ἑκατέραν τῶν διαζευχθεισῶν, ὅταν καὶ αὗται δηλονότι ληφθῶσιν ἐτέρων γεννητικαὶ ἐν τῇ κατ' ὀρθὸν ἢ κατ' ἀναστροφὴν θέσει, τὸ αὐτὸ συνενεχθήσεται. οἷον ἐπὶ τῆς ἐν τριπλασίῳ λόγῳ σχέσεως τῆς α γ θ, ὀρθῶς μὲν οὕτω κειμένης ὡς ἀνεφύη ὁ ἔσχατος τετράγωνος αὐτῆς (ἡγουν ὁ ἐλάσσων ὅρος) ἢ μονὰς ὁ αὐτὸς μένει καὶ ἐν τῇ ἐξ αὐτῆς ἀποτεχθείσῃ τετραπλασίῳ τῇ α δ ις, τουτέστιν ἐλάττων τε καὶ ὑπόλογος· ἀντιστρόφως δὲ ληφθείσης τῆς αὐτῆς τριπλασίου σχέσεως, τουτέστι {θ γ} α, ὁ πρῶτος ταύτης τετράγωνος (ἡγουν ὁ μείζων ὅρος) ὁ θ εἰς τὸν ἐλάττονα μεταβαίνει ὅρον καὶ ἀντὶ προλόγου γίνεται ὑπόλογος ἐν τῇ ἐξ αὐτῆς ἀποτελουμένη ἐπιτρίτῳ σχέσει τῇ θ ιβ ις. ὡσαύτως καὶ ἐπὶ τῆς ἐτέρας τῶν διαζευχθεισῶν τῆς δ ς θ ἡμιολίου σχέσεως ὁ ἔσχατος τετράγωνος· αὐτῆς δὴ τῆς ἡμιολίου σχέσεως (ἡγουν ὁ ἐλάττων ὅρος) ὁ δ ὁ αὐτὸς μένει (ἡγουν ἐλάττων καὶ ὑπόλογος) καὶ ἐν τῇ ἐξ αὐτῆς ἀποτελεσθείσῃ κατὰ διπλασιεφημιόλιον λόγον σχέσει τῇ δ ι κε (ὀρθῶς δηλονότι κειμένης – οὕτως ὡς ἀνεφύη – ἀπὸ τῆς ἐν διπλασίονι λόγῳ ἀνεστραμμένης σχέσεως τῆς δ β α), ἀνεστραμμένης δὲ ληφθείσης τῆς αὐτῆς ἡμιολίου, τουτέστι θ ς δ, ὁ πρῶτος ταύτης (ἡγουν ὁ μείζων ὅρος) ὁ θ, εἰ καὶ ὁ αὐτὸς μένει καθὼς τετράγωνος, ἀλλ' οὖν εἰς τὸν ἐλάττονα μεταβαίνει ὅρον καὶ ἀντὶ προλόγου γίνεται ὑπόλογος ἐν τῇ ἐξ αὐτῆς ἀποτεχθείσῃ ἐπιδιμερεῖ (ἡγουν δισεπιτρίτῳ) σχέσει τῇ θ ιε κε.

πάλιν κατὰ δευτέραν, ὡς οὕτω φάναι, ὑποδιαίρεσιν ἐκάστης τῶν διεζευγμένων, ὅταν δηλονότι καὶ αὗται ληφθῶσιν ἐτέρων γεννηταὶ ἐν τῷ ὀρθῶς κείσθαι ἢ ἀνεστράφθαι, τὸ αὐτὸ συμβήσεται. χάριν δὲ σαφηνείας ἐν σκέλος προκείσθω μία τῶν δηλωθεισῶν διεζευγμένων· ἔστω δὲ τοῦτο λόγου χάριν τῆς δισεπιτρίτου σχέσεως. τεχθήσεται οὖν ἀπὸ ταύτης ὀρθῶς μὲν κειμένης ἀπὸ τοῦ ἐλάττονος ὅρου ἢ διπλασιεπιδίτριτος, ἀνεστραμμένης δὲ ἐκ τοῦ μείζονος ἢ τρισεπίπεμπος ὡς ἐκ τῆς θ ιε κε ἢ θ κδ ξδ ἢ κε μ ξδ. καὶ ἐπὶ τῶν ἄλλων ὡσαύτως σχέσεων ἢ αὐτὴ φανήσεται πρόοδος, ἐν μὲν τῇ κατ' ὀρθὸν σχέσει τῇ γε{ννητικῇ τοῦ} ἐσχάτου τετραγώνου (ἡγουν τοῦ ἐλάττονος ὅρου ταύτης τῆς κατ' ὀρθὸν σχέσεως) τοῦ αὐτοῦ μένοντος (ἡγουν ἐλάττονος {καὶ ὑπολόγου} φυλαττομένου) καὶ ἐν τῇ ἐξ αὐτῆς ἀπογεννωμένη σχέσει, ἐν δὲ τῇ κατ' ἀναστροφὴν σχέσει τῇ {γεννητικῇ} τοῦ μείζονος ὅρου καὶ προλόγου ταύτης τῆς ἀνεστραμμένης σχέσεως μεταβαίνοντος ἐν τῇ ἐξ αὐτῆς {ἀποτεχθείσῃ εἰς τὸν} ἐλάττονα ὅρον καὶ ὑπόλογον.

οὐ λέληθε δέ με ὡς ὁ ἀριστοτελικώτατος Δυρραχίου ἄλλως {ἐρμηνεύει τὸ ῥητόν,} ὡς ὁ ἔσχατος καὶ τελευταῖος ἀριθμὸς τῶν διεζευγμένων ([ἦτοι s.l.] τῶν ἐκ τῆς αὐτῆς καὶ μιᾶς σχέσεως ἀπο{γεννωμένων} σχέσεων) εὑρεθήσεται, ὡς ἀπὸ τῆς α β δ ἢ α γ θ ἢ δ ς θ. ἀλλὰ τούτου τὸν λόγον ὡς κεκομψευμένον μὲν ἐπαινώ, εἰ δὲ ση{μαίνει τὴν διάνοιαν} τοῦ κειμένου, συνιδεῖν οὐκ ἔχω· ἢ γὰρ ἂν, εἰ κατὰ τοῦτον τὸν νοῦν ἐξελήφθη τῷ Νικομάχῳ τὸ ῥητόν, οὐκ ἂν εἶπεν “ὁ μὲν ἔσχατος τετράγωνος ὁ αὐτὸς μήναι”, ἀλλ' ἀντὶ τοῦ “μέναι” εἶπεν [[τὸν]] “εὑρεθήσεται”.

*Glosses in Marc. gr. Z. 318**Ar. I.23.15*

ἐπὶ πασῶν {s.l. σχέσεων δηλονότι ἐπιμερῶν καὶ πολλαπλασιεπιμορίων} δὲ τῶν διαzeugθεισῶν {s.l. τῶν προλεχθεισῶν σχέσεων} καὶ ἀφ' ἧς {s.l. σχέσεως δηλονότι τῆς ἐπιμορίου} ἀμφοτέραι, ὁ μὲν ἔσχατος τετράγωνος ὁ αὐτὸς μένει, ὁ δὲ πρῶτος {s.l. λόγος} εἰς τὸν ἐλάττονα μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

Adjunct

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοί εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ' ἀλλήλαις γενομένων, καὶ ὁ μὲν πρῶτος {s.l. τουτέστιν ὁ γεννῶν αὐτὸν ἀπαιστοῦ γεννωμένου ἐλάττων γίνεται τῷ ποσῷ m. rec.} τῆς ἀπογεννώσεως εἰς τὸν ἐλάττονα τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ μείζων τετράγωνος ἐστίν

C.9. Four scholia in Vat. gr. 256, f. 316r-v [abcd], Ambros. I 8 sup., ff. 33r-34r [abcd], and Athens, EBE 1115, pp. 58-59 [abcd], Oxon. Bodl. Selden Supra 20, ff. 16v-17r [bd], Par. gr. 2479, f. 92r (m. rec.) [bd], Vat. gr. 186, f. 243r [a]

In Ambros. I 8 sup. and Vat. gr. 256, the scholia are anchored on the indicated words; in Vat. gr. 186, the reference sign is located at the beginning of Ar. I.23.15; anchoring and reference signs in Athens, EBE 1115 are the same as those of Vat. gr. 256. The quotation of Ar. I.23.15 is in red. Cf. Texts C.7a and C.8a for the reading in blue in Text a.

a. Ἐφεσίου φιλοσόφου σχόλιον

<πασῶν.> διαzeugθείσας λέγει τὴν μονάδα καὶ δυάδα καὶ τετράδα τὰς ἐκ τῶν τριῶν μονάδων γεννηθείσας κατὰ τὸ πρόσταγμα τὸ λέγον “πρῶτον πρώτῳ ἴσον ποιήσον”, καὶ πάλιν τὰς γεννηθείσας ἀπ' αὐτῶν τῆς τε μονάδος καὶ δυάδος καὶ τετράδος, αἰτινές εἰσι μία, τρεῖς, θ, καὶ τὰς ἐκ τῆς ἀντιστροφῆς αὐτῶν τῆς δ β α, αἰτινές εἰσι δ ς θ. ἐπὶ πασῶν οὖν τούτων τῶν διαzeugθεισῶν καὶ ἀφ' ἧς πρώτης διαzeugθεισῶς, τουτέστι τῶν τριῶν μονάδων, ὁ μὲν ἔσχατος τετράγωνος (οἷον ὁ δ) ὁ αὐτὸς μένει, ὁ δὲ πρῶτος (ἦτοι ἡ μονάς· καὶ γὰρ αὕτη πάντα δυνάμει) εἰς τὸν ἐλάττονα μεταβαίνει, ἦτοι τὸν θ· ὁ γὰρ θ ἡμιόλιος, ὁ δὲ ἡμιόλιος λόγος ἐλάττων τοῦ διπλασίου, ὥσπερ καὶ ὁ ἐπίτριτος τοῦ ἡμιολίου.

Ambros. I 8 sup.: om tit. | –περ καὶ — ἡμιολίου legi nequit | Athens, EBE 1115: δυάδα καὶ τριάδα | Vat. gr. 186: om tit. | ἔννεα | δ β β | τουτέστιν ὁ δ | εἰς τὸν ἐλάττω μεταβαίνει

b. <διαzeugθεισῶν.> διαzeugθείσας λέγει τὰς ἀφ' {ἐφ' 1115} ὅπου οὖν τῶν ἐπιμορίων σχέσεις γεννηθείσας {–ήσας 1115}. οἷον ἀπὸ τῆς {τοῦ 1115} ἡμιολίου τοῦ δ {τοῦ add. 1115} ς θ ὁρθῶς μὲν κειμένης ἐγένετο τῶν πολλαπλασιεπιμορίων ὁ διπλασιεφήμισος ὁ δ ι κε, ἀντεστραμμένως {–ων 1115 : comp. 256} δὲ ἐγένετο τῶν {def. 1115} ἐπιμερῶν ὁ ἐπιδίτριτος ὁ θ ιε κε. τὸ μὲν οὖν “διαzeugθεῖσαι” εἴρηται περὶ τῶν δύο σχέσεων τῶν ἀπὸ τοῦδε τοῦ

ἐπιμορίου, οἷον εἰ τύχοι τοῦ ἡμιολίου γενομένων. τὸ δὲ “ἀφ’ ἧς ἀμφοτέραι” περὶ αὐτῆς τῆς {τῆς om. 20 2479} ἐπιμορίου ἀφ’ ἧς αἱ διαζευχθεῖσαι γεγόνασιν. φησὶ οὖν ὅτι ἐν {ἐν om. 2479} ἀμφοτέραις ταῖς διαζευχθείσαις εἷς ἐστὶ καὶ ὁ αὐτὸς {ὁ add. 20 2479} ἔσχατος {τε καὶ add. 20 2479} τετράγωνος, ὥσπερ ἐν τῇ δισηπιδίτῳ τῇ θ ιε κε καὶ τῇ διπλασιεφημίσει τῇ δ ι κε {ιε 20 2479} εἷς ἐστὶν ὁρος ἔσχατος {ἔ. ὁ. 20 2479} ὁ κε· ὁ δὲ τῆς ἀπογεννησάσης ταύτας τὰς σχέσεις ἡμιόλιος {ἡμιολίου 20 2479} πρῶτος τετράγωνος (ἡγουν ὁ μείζων αὐτῆς ὁ θ· τοῦτον γὰρ ἀνωτέρω πρῶτον ἔθηκεν ὁ Νικόμαχος), οὗτός φησιν {seq. μείζων ὧν hic hab. 20 2479} ἐν τῷ ἡμιολίῳ τῷ δ ς θ μείζων ὧν ἡττων γίνεται ἐν τῷ δισηπιδίτῳ τῷ θ ιε κε· ἐν ταύτῃ γὰρ τῇ σχέσει καὶ {καὶ om. 20 2479} ὁ θ ἡττων γίνεται ἐν {δὲ add. 20 2479} τῇ ἡμιολίῳ μείζων ὧν {γίνεται 20 2479}. καὶ ἐπὶ τῶν λοιπῶν δὲ σχέσεων εὐρήσεις τὸ ἀνάλογον.

Oxon. Bodl. Selden Supra 20 & Par. gr. 2479: ἀφοτοῦν | γεννηθείσας σχέσεις | ἀντεστραμμένον | τῷ δ καὶ θ 2479 | add. in fine e Rec. I διαζευχθεῖσαι δὲ εἰσι σχέσεις αἱ τε κατ’ ὀρθὸν ἐπιζευγνύμεναι καὶ αἱ κατὰ ἀναστροφὴν συμπλεκόμεναι· διέζευκται γὰρ τοῦ ὀρθοῦ τὸ ἀντεστραμμένον

c. <τετράγωνοι.> ἐδείχθη παρὰ τῷ γεωμέτρῃ ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ὥσι πρὸς ἀλλήλους τὸν αὐτὸν λόγον ἔχοντες, οἱ ἄκροι αὐτῶν τετράγωνοί εἰσιν.

Ambros I 8 sup.: a tineis voratus

To the Adjunct

d. <μέσοι.> οἱ μέσοι φησὶ τῶν ἐκκειμένων ἐκθέσεων ἐκ τῶν πλευρῶν αὐτῶν τῶν ἄκρων γίνονται ἐπ’ ἀλλήλαις γενομένων. οἷον ἐπὶ τῶν ἡμιολίων τοῦ δ ς θ ὁ ς μέσος ὧν ἐκ τῶν πλευρῶν τῶν ἄκρων γίνονται, τουτέστι τοῦ δ καὶ τοῦ θ· πλευρὰ γὰρ τοῦ δ ἡ δυάς, καὶ πλευρὰ τοῦ θ ἡ τριάς, αἵτινες ἀπ’ ἀλλήλαις γινόμεναι ποιοῦσι τὸν μέσον τὸν ς· δις γὰρ γ, ς, καὶ ἐπὶ τῶν ἄλλων ὁμοίως εὐρήσεις ταῦτα. πλευρὰ τοῦ κε τὰ ε, τοῦ πα τὰ θ· ἐκ τῶν πλευρῶν τῶν δύο τοῦ πα καὶ τοῦ κε (λέγω δὲ τοῦ ε καὶ {τοῦ add. 1115} θ· ταῦτα γὰρ αἱ πλευραὶ) γίνεται ὁ μέσος ὁ με τῶν πλευρῶν ἐπ’ ἀλλήλαις γενομένων· πεντάκις γὰρ θ, με. πάλιν πλευρὰ τοῦ κε τὰ ε, τοῦ δὲ ρος τὰ ιδ· τεσσαρισκαιδέκατον γὰρ τοῦ ρος τὰ ιδ· τεσσαρισκαιδέκατον γὰρ ιδ, ρος. ἢ πλευρὰ οὖν τοῦ κε (τὰ ε λέγω δὴ) καὶ ἡ πλευρὰ τοῦ ρος (λέγω δὴ τὰ ιδ) συνελθόντα πρὸς ἄλληλα ἀποτελοῦσι τὸν ο· πεντάκις γὰρ τὰ ιδ γίνονται ο.

Oxon. Bodl. Selden Supra 20 & Par. gr. 2479: τοῦ δ, τοῦ ς καὶ τοῦ θ, καὶ {καὶ om. 20} ὁ ς | γίνεται | τὸν ς om. 20 | ἐπ’ ἀλλήλαις | des. εὐρήσεις {ταῦτα 20}

C.10. Isaak Argyros’ glosses and scholium in Norimb. Cent. V. App. 36, f. 18r

A sign anchors the scholium on the underlined word; the infralinear glosses are also transcribed.

Ar. I.23.15

ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν {s.l. ἡγουν τῶν ἀπὸ μιᾶς τινὸς σχέσεως ἀπογεννηθεισῶν} καὶ ἀφ’ ἧς ἀμφοτέραι {s.l. ἡγουν δύο}, ὁ μὲν ἔσχατος {s.l. ἡγουν ὁ ὑπόλογος τῆς

ἀπογεννώσης} τετράγωνος ὁ αὐτὸς {s.l. ἤγουν ὑπόλογος} μένει {s.l. ἤγουν ἐν τῇ μιᾷ δηλονότι}, ὁ δὲ πρῶτος {s.l. ἤγουν ὁ πρόλογος} εἰς τὸν ἐλάττονα {s.l. ἤγουν τὸν ὑπόλογον τῆς ἐτέρας} μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

διαzeugθείσας φησὶ τὰς ἀπὸ μιᾶς τινος σχέσεως ἀπογεννηθείσας ἐτέρας β σχέσεις τὴν τε δηλονότι κατ' ὀρθότητα καὶ τὴν κατ' ἀντιστροφὴν, αἱ καὶ διεzeugνυνται πάντως ἀλλήλων, ὅτι ἢ μὲν κατ' ὀρθότητα τοῦ αὐτοῦ γένους ἐστὶ τῇ ἀπογεννώσει ἢ δὲ κατ' ἀντιστροφὴν ἐτέρου γένους. καὶ φησιν ὅτι ὁ μὲν ἔσχατος τῆς ἀπογεννώσεως, ἦτοι ὁ ὑπόλογος, τετράγωνος ὢν ὁ αὐτὸς διαμένει, τουτέστιν ὑπόλογός ἐστι καὶ ἐν τῇ ἐξ αὐτῆς ἀπογεννηθείσῃ κατ' ὀρθότητα, ὁ δὲ πρῶτος, ἤγουν ὁ πρόλογος, τῆς ἀπογεννώσεως ἐπὶ τὸν ἐλάττονα (ἤγουν τὸν ὑπόλογον) τῆς κατ' ἀντιστροφὴν ἀπογεννώσεως διαβαίνει. οἷον ἐπὶ ὑποδείγματος ἔστω σχέσις διπλασία ἢ α β δ, καὶ γιγνέσθω διὰ τῶν προσταγμάτων κατὰ μὲν ὀρθότητα ἐτέρα σχέσις τριπλασία ἢ α γ θ, κατὰ δὲ ἀντιστροφὴν ἡμιολία ἢ δ ς θ. ὅρα ὅτι ἡ μονὰς ὑπόλογος οὔσα ἐν τῇ διπλασίᾳ ὑπόλογός ἐστι καὶ ἐν τῇ τριπλασίᾳ κατ' ὀρθότητα ἀπογεννηθείσῃ, ὁ δὲ δ πρόλογος ὢν ἐν ἐκείνῃ ὑπόλογος γέγονεν ἐν τῇ ἡμιολίᾳ· κατ' ἀντιστροφὴν γάρ. ὅ τι δὲ φησιν, ὡς οἱ ἄκροι τῶν τοιούτων σχέσεων τετράγωνοί εἰσιν, οὐκ ἐπὶ πασῶν φαίνεται συμβαίνειν, ἀλλ' ἐπὶ μόνων τῶν ἀπ' ἰσότητος, ἥτις ἐστὶν ἐν μονάσιν ἢ τετράσιν ἢ ἄλλοις τισὶ τετραγώνοις οὖσιν ἀριθμοῖς· αἱ γὰρ ἀπὸ δυάδων ἢ τριάδων ἢ ἄλλου τινὸς μὴ τετραγώνου ἀριθμοῦ οὐκ ἔχουσι τοὺς ἄκρους τετραγώνους. οἷον ἔστω β β β· πάντως κατὰ τὰ προστάγματα γενήσονται ἢ τε β δ η, καὶ ἐκ ταύτης ἢ τε β ς ιη καὶ ἢ η ιβ ιη, ἀλλ' οὐχ ἔξουσι τοὺς ἄκρους τετραγώνους. αἴτιον δ' ὅτι αἱ ἀπὸ μονάδων ἐλάχισται εἰσι τῶν τὸν αὐτὸν λόγον ἔχόντων αὐταῖς, καὶ δέδεικται τῷ στοιχειωτῇ ὅτι ἐὰν ὦσι τρεῖς ἀριθμοὶ ἐλάχιστοι τῶν τὸν αὐτὸν λόγον ἔχόντων αὐτοῖς, οἱ ἄκροι τετράγωνοί εἰσιν.

C.11. Theodoros/Demetrios Protocensor's scholium

The distinctive differences between the two versions of the scholium are in red; note, however, the variant reading shared by the family of Roma, Arch. S. Paolo 24C and Version **b**. In Version **b**, the manuscripts marked in blue only contain Rec. I.

a

Bucur., BAR gr. 620, p. 224; Gott. philol. 66, f. 141r; Laur. Plut. 28.35, ff. 29v-30r; Monac. gr. 482, f. 220r; Par. gr. 2480, pp. 345-346; Roma, Arch. S. Paolo 24C, f. 55r

σχόλιον {σχόλ^a 620 66 482 24C} Θεοδώρου πρωτοκένσωρος
 ἔσχατον ἐνταῦθα τετράγωνον τὸν αὐτὸν μένοντα τὸν ὑπόλογον λέγει {λέγ^v 66 2480 24C : -ω 28.35 482}· αὐτὸς γὰρ ἐν πάσαις ταῖς σχέσεσιν ὑπόλογος μένει, ἐν τε {οὔτε 620 66 2480} τῇ ὀρθῇ ἐκθέσει καὶ τῇ ἐξ ἀντιστροφῆς, οἷον χάριν σαφηνείας ἐν τῇ ὀρθῇ ἐκθέσει ἐγεννήθη ἀπὸ τῶν ἐξ ἀντιστροφῆς διπλασιῶν (ἤγουν τοῦ δ καὶ τοῦ β καὶ τοῦ α) ὃ τε δ καὶ ὁ ς καὶ ὁ θ ἡμιόλιοι. ἀντίστροφον τούτους, καὶ ἔσονται θ ς {καὶ add. 620 66 2480 24C} δ.

ιδου ἐν ταῖς δυσὶ ταύταις ἐκθέσεσι (τῇ ὀρθῇ φημι καὶ τῇ ἐξ ἀντιστροφῆς) ὁ δὲ ὑπόλογος εὐρίσκεται ὁ δὲ θ πρόλογος· ἡμιόλιος γὰρ αὐτοῦ τοῦ ζ ἐστίν. ἐν δὲ τῇ γεννηθείσῃ ἀπὸ τῆς ἐξ {ἐξ om. 620 66 2480} ἀντιστροφῆς ἡμιολίου ἐκθέσεως ὁ πρόλογος (ἡγουν ὁ θ), ὅστις {ὅς ἐστίν 620 66 2480 24C}, ὡς {ὡς om. 28.35} εἶπομεν, ἡμιόλιος ἐστὶ {ἐστὶ om. 620 66 2480 24C} τοῦ ζ, ὑπόλογος εὐρίσκεται ἐν ἐπιμερεῖ σχέσει {σχει 66} τοῦ ιε. οἷόν τί φημι ἦσαν ἐκ {ἐν 620 66 2480 24C} τῇ ἐξ ἀντιστροφῆς ἡμιολίου ἐκθέσεως {ἡμιο^λ ἐκθ/ 24C} θ ζ δ· ἐκ τούτων {-του 620 66 2480 : τοῦ^τ 24C} ἐγεννήθησαν θ ιε κε. ἐνταῦθα ὁ ἐν ἐκείνοις θ ἡμιόλιος ὦν τοῦ ζ καὶ μείζων ὡς πρόλογος {-οι 620 66 : πρόλο^γ 24C} εἰς τὴν τοῦ {τοῦ om. 28.35} ἐσχάτου τάξιν μεταβαίνει, καὶ γίνεται ὑπόλογος ὑπεπιμερῆς τοῦ ιε. καὶ τοῦτο ἀκριβῶς σκοπῶν ἐπὶ πασῶν τῶν σχέσεων οὕτως εὐρήσεις {εὐρήσεις om. 2480}.

Par. gr. 2480: perperam add. in fine ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοι εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ' ἀλλήλαις γενομένων, καὶ ὁ μὲν πρῶτος τῆς ἀπογεννώσεως εἰς τὸν ἐλάττονα τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ μείζων τετράγωνος ὁ αὐτός ἐστιν

b

Ambros. I 83 inf., f. 89r (m. rec.); Angel. gr. 1, f. 33r; BNCF, Fondo Naz. (olim Magliab.) II.III.37, ff. 45v-46r; Marc. gr. Z. 333, f. 61v; Monac. gr. 76, f. 158v; Mutin. α.T.8.14, f. 27v; Mutin. α.U.9.7, f. 38r; Par. Coislin 174, f. 102r-v; Par. gr. 2531, f. 79v; Vat. gr. 2262, f. 74r-v

σχόλιον {σχό^λ 1 α.U.9.7 174 2531 : σχόλια 37 83} Δημητρίου πρωτοκένσωρος {-σορ- 83 : -ντο- 2531} {εἰς τὰ αὐτὰ solum 333 : tit. om. 76 α.T.8.14 2262}

ἔσχατον ἐνταῦθα τετράγωνον τὸν αὐτὸν μένοντα τὸν ὑπόλογον λέγει· αὐτὸς γὰρ ἐν πάσαις ταῖς σχέσεσιν [[τὸν 37]] ὑπόλογος {-ον 2531 : -ο dein ras. 37} μένει, ἔν τε τῇ ὀρθῇ ἐκθέσει καὶ τῇ ἐξ ἀντιστροφῆς {ἀνα- 76 174}, οἷον χάριν σαφηνείας ἐν τῇ ὀρθῇ ἐκθέσει ἐγεννήθη ἀπὸ τῶν ἐξ ἀντιστροφῆς διπλασίων {τοῦ ἐ. ἀ. διπλασίου 76} (ἡγουν τοῦ δ καὶ τοῦ β καὶ τοῦ α) ὃ τε {τε om. 333 : τὰ in ras. m. rec. α.T.8.14} δ καὶ ὁ ζ καὶ ὁ θ {ὁ δ ζ θ 333} ἡμιόλιοι {-ος 1 2262}. ἀντίστρεψον τούτους {-οις 2531}, καὶ ἔσσονται θ ζ καὶ {καὶ om. 76 174} δ. ἰδοὺ ἐν ταῖς δυσὶ ταύταις ἐκθέσεσι (τῇ ὀρθῇ φημι καὶ τῇ ἐξ ἀντιστροφῆς) ὁ {μὲν add. 333 : add. m. rec. α.T.8.14} δ ὑπόλογος εὐρίσκεται ὁ δὲ θ πρόλογος {-ον 2531}· ἡμιόλιος γὰρ αὐτοῦ τοῦ ζ ἐστίν. ἐν δὲ τῇ γεννηθείσῃ ἀπὸ τῆς ἐξ ἀντιστροφῆς {τοῦ add. 333 : add. m. rec. α.T.8.14} ἡμιολίου {ex -ω 37} ἐκθέσεως ὁ πρόλογος (ἡγουν ἥως 2531) ὁ θ), ὅς ἐστιν, ὡς εἶπομεν, ἡμιόλιος τοῦ ζ, ὑπόλογος εὐρίσκεται ἐν ἐπιμερεῖ σχέσει τοῦ ιε. οἷόν τί φημι ἦσαν ἐν τῇ {τῇ om. 76} ἐξ ἀντιστροφῆς ἡμιολίῳ ἐκθέσει θ ζ δ· ἐκ τούτου {τούτων 333 : m. rec. α.T.8.14} ἐγεννήθησαν θ ιε κε. ἐνταῦθα ὁ ἐν ἐκείνοις θ ἡμιόλιος ὦν τοῦ ζ καὶ μείζων ὡς πρόλογος εἰς τὴν τοῦ ἐσχάτου τάξιν μεταβαίνει, καὶ γίνεται ὑπόλογος ὑπεπιμερῆς τοῦ ιε. καὶ τοῦτο ἀκριβῶς σκοπῶν ἐπὶ πασῶν τῶν σχέσεων οὕτως {-ων 83} εὐρήσεις.

C.12. Glosses, and scholia to the Adjunct, in Hamburg, S-UB, philol. 88, ff. 23v-24r, and Vat. gr. 2387, f. 48v; cf. Marc. gr. Z. 317, f. 23r-v, Marc. gr. Z. 595, f. 32r-v, Par. gr. 2762, f. 40v

A sign in the Hamburg manuscript anchors each scholium on the underlined word; the infralinear glosses in Hamburg, S-UB, philol. 88 and in Marc. gr. Z. 317 are also transcribed.

Ar. I.23.15

ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν {διαλεχθεισῶν καὶ διαζευχθεισῶν 317 : τοῦ τριπλασίου καὶ τοῦ ἐπιμορίου καὶ τῆς σχέσεως αὐτῆς τῆς διπλασίου marg. 317} καὶ ἀφ' ἧς ἀμφοτέραι, ὁ μὲν ἔσχατος τετράγωνος μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα {s.l. κατὰ τὴν σχέσιν· τὸ γὰρ πρότερον ὑπόκειται ἡμιολίον, τὸ δεύτερον ἐπίτριτον 88} μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

Adjunct

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντες οἱ ἄκροι τετράγωνοι εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ' ἀλλήλαις γενομένων, καὶ ὁ μὲν πρῶτος {marg. ἡγουν ὁ μείζων ὅρος 317} τῆς ἀπογεννώσεως εἰς τὸν ἐλάττονα {s.l. κατὰ τὴν σχέσιν 88} τῆς γινομένης {marg. ἡγουν τῆς ἐξ ἀναστροφῆς γινομένης 317} μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ μείζων τετράγωνος ὁ αὐτός ἐστιν.

a. συστοιχία πρώτη ἐστὶ τὸ α, τὸ {τὰ 317} β, τὸ δ· αὕτη γὰρ ἀπογεννᾷ τὴν ὑποκάτω αὐτῆς, ὡς καὶ αἱ λοιπαὶ τὰς ὑποκάτω αὐτῶν. ὁ γοῦν πρῶτος τῆς ἀπογεννώσεως ταύτης συστοιχίας μεταβαίνει εἰς τὸν ἐλάσσονα τῆς γινομένης (ἦτοι εἰς τὸν θ)· ὡς ἔσχατος γὰρ λέγεται {λέγ' 88} {ὁ add. 317} ἐλάσσων. καὶ ἐπὶ τῶν ἄλλων οὕτως ὁμοίως.

Marc. gr. Z. 595 & Par. gr. 2762: α β δ | ὑπογεννᾷ | ὡς ὁ ἔσχατος (cum 317)

b. οἶον ὁ δ καὶ ὁ θ εἰσὶ τετράγωνοι. πλευρὰ γοῦν τοῦ {τῶν 2387} δ εἰσὶ τὰ β, τοῦ {τῶν 2387} δὲ θ πλευρὰ τὰ γ, ἅτινα (ἡγουν τὰ β καὶ γ) μιχθέντα ἐγένοντο ζ· δις γὰρ τρίς, ζ, ὅστις καὶ οὗτος τετράγωνος ἐστὶν ἀλλ' ἀνισόπλευρος διὰ τὸ τὴν μίαν πλευρὰν διαφέρειν τῆς ἐτέρας μονάδι. ὡσαύτως ὁ θ καὶ ὁ ις εἰσὶ τετράγωνοι. πλευρὰ γοῦν τοῦ θ εἰσὶ τὰ γ, τοῦ δὲ ις τὰ δ, ἅτινα (ἡγουν τὰ τρία καὶ τὰ τέσσαρα) μιγέντα ἐποίησαν τὸν μέσον αὐτῶν, ἦτοι τὸν ιβ· τρίς γὰρ δ, ιβ, ὅστις καὶ οὗτος τετράγωνος λέγει ἀλλ' ἀνισόπλευρος διὰ τὸ τὴν μίαν πλευρὰν ὑπερέχειν τῆς ἐτέρας μονάδι. καὶ ἐπὶ πασῶν οὕτω γίνεται, ἡγουν ἐκ τῶν πλευρῶν τῶν ἄκρων γίνονται οἱ μέσοι.

Marc. gr. Z. 317 & Marc. gr. Z. 595 & Par. gr. 2762: om. δὲ (pr.) | οὕτως (οὗτος 317) | des. τετράγωνος ἐστὶν

C.13. Dishypatos' and the one from Dirrachios' (sic) scholia in Vat. gr. 198, f. 19r, and Hamburg, S-UB, philol. 89, pp. 368-369

a. εἰς τοῦτο ὁ μὲν Δισύπατος Νικήτας λέγει ὅτι διαζευχθείσας λέγει ὁ Νικόμαχος τὴν τε ἐξ ὀρθῆς καὶ τὴν ἐξ ἀντεστραμμένης γεννηθείσας ἀπὸ τῆς μιᾶς καὶ πρώτης σχέσεως. λέγει γοῦν ὅτι ὁ μὲν ἔσχατος οἰονεῖ ὑπόλογος ὁ καὶ ἀρχικώτερος ὁ αὐτὸς μένει ἐν τῇ ἐξ ὀρθῆς ἀπογεννηθείσῃ σχέσει, ὁ δὲ πρῶτος εἰς τὸν ἐλάσσονα μεταβαίνει, δηλαδή ἐν τῇ ἐξ ἀντεστραμμένης ἀποκυηθείσῃ σχέσει· γίνεται γὰρ ὁ πρόλογος ὑπόλογος. οἷον ἐστὶν α β δ διπλασία σχέσις. ἐκ ταύτης ὀρθῆς μὲν γεννᾶται ἡ α γ θ τριπλασία σχέσις, ἀντεστραμμένης δὲ ἡ δ ζ θ ἡμιόλιος σχέσις. ὅρα οὖν τὸν ἐν τῇ διπλασίᾳ σχέσει ἔσχατον (οἰονεῖ τὸν α) ὅτι ἐν τῇ σχέσει τῇ ἐχούσῃ αὐτὸν – δηλαδή τῇ τριπλασίᾳ· ἐν γὰρ τῇ ἡμιολίᾳ οὐκ ἔστι – μένει ὁ αὐτός, δηλαδή ὑπόλογος. ὅρα καὶ τὸν πρῶτον ἐν τῇ τοιαύτῃ σχέσει τῇ διπλασίᾳ (οἰονεῖ τὸν δ) ὅτι ἐν τῇ σχέσει τῇ ἐχούσῃ αὐτόν (ἡγουν τὸν δ) – δηλαδή τῇ ἡμιολίᾳ· ἐν γὰρ τῇ τριπλασίᾳ οὐκ ἔστιν – οὐκέτι μένει ὁ αὐτός· ἐκεῖσε γὰρ ἐν τῇ διπλασίᾳ σχέσει πρόλογος ἦν· ἐνταῦθα δὲ ἐν τῇ ἡμιολίᾳ εἰς τὸν ἐλάσσονα μεταβαίνει, οἰονεῖ ὁ πρόλογος ἐγένετο ὑπόλογος, ἔσχατον καὶ ἐλάσσονα λέγων τὸν ὑπόλογον πρῶτον δὲ τὸν πρόλογον.

b. ὁ δὲ Διρραχίου φησὶν ὡς σκόπησον τὰς διεζευγμένας σχέσεις ἀπὸ τῆς α^{ης}, οἷον ἀπὸ διπλασίου α β δ ὀρθῆς μὲν τριπλασίαν α γ θ ἀντεστραμμένης δὲ ἡμιολίαν δ ζ θ. ὁ ἔσχατος τοίνυν καὶ ἐπὶ τῇ τριπλασίᾳ καὶ ἐπὶ τῇ ἡμιολίᾳ ὁ αὐτὸς μένει (ἡγουν ὁ θ), ὁ δὲ ἐν τῷ τριπλασίῳ α^{ος} (ἡγουν ὁ α) καὶ ἐν τῇ ἡμιολίᾳ πρῶτος (ἡγουν ὁ δ) εἰς τὸν ἐλάχιστον μεταβαίνει, ἡγουν ἐν τῇ διπλασίᾳ ἦν ὁ α ὑποδιπλάσιος ἐνταῦθα δὲ γέγονε ὑποτριπλάσιος, ὅς ἐστιν ἔσχατος τοῦ ὑποδιπλασίου. καὶ πάλιν ὁ δ ἐκεῖσε μὲν ἐν τῇ διπλασίᾳ ἦν πρόλογος καὶ μείζων τῶν ἄλλων ἀριθμός, ἐνταῦθα δὲ ἐν τῇ ἡμιολίᾳ καὶ ὑπόλογος καὶ ἐλάχιστος ἀριθμός· ἐλάχιστος γὰρ ὁ δ τοῦ ζ ἢ τοῦ θ.

C.14. Glosses and a scholium in Marc. gr. Z. 320, f. 52r-v (Gregoras' hand), Par. gr. 2450, f. 111r, Vindob. phil. gr. 220, ff. 61r-62r, Neapol. III.C.1, f. 20v, Vat. Barb. gr. 273, f. 122r-v [a]

A sign anchors the scholium on the underlined word in Par. gr. 2450 and Vat. Barb. gr. 273; the infralinear glosses of these two manuscripts and of Vindob. phil. gr. 220 are also transcribed; Neapol. III.C.1 does not have infralinear glosses; all glosses and scholia stem from Marc. gr. Z. 320, which only anchors the first scholium; the sign is also found in the accompanying table (f. 51v), followed by ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν. Quotations of Ar. I.23.15 are in red.

Ar. I.23.15

ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν {s.l. τῶν προλεχθεισῶν σχέσεων 220 : ἐπιμορίων καὶ ἐπιμερῶν 2450 273 : τῶν προλεχθησῶν σχέσεων 220} καὶ ἀφ' ἧς {s.l. τῆς πολυπλασίου

2450 273 : non habent 320 220} ἀμφότεραι, ὁ μὲν ἔσχατος τετράγωνος {s.l. ὁ ἄκρος ἑκάτερος 320 2450 273 : ὁ ἄκρος 220} ὁ αὐτὸς μένει {s.l. ἀμετάβατος μένει 320 2450 273 : ἀμετάβατος 220}, ὁ δὲ πρῶτος {s.l. ἡγουν ὁ μείζων λόγος 320 2450 273 : non habet 220} εἰς τὸν ἐλάττονα {s.l. ἡγουν τὸν ἐπιμόριον καὶ τὸν ἐπιμερῆ· ἡττων γὰρ οὗτος τοῦ πολλαπλασίου 320 2450 220 : — ἐπιμερῆ· ἡγουν *τρίτον* 273} μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

a. *πᾶσαι αἱ διαζευχθεῖσαι σχέσεις* εἰσὶν αἱ ἐπιμόριοι καὶ ἐπιμερεῖς· ἡ δὲ *ἀφ' ἧς* {*διαφ' ἧς* III.C.1} *ἀμφότεραι* ἔστιν ἡ πολλαπλασίος, ἃς πάσας ὡς ὁρᾷς ἐξεθέμεθα {*ἐν τῷ ἀριστῷ μέρει* 320 III.C.1 220}, {inc. alia m. 320} ὧν *πάντες οἱ ἄκροι τετράγωνοι*. ὁ μὲν οὖν {*οὖν* om. III.C.1 220} *ἔσχατος τετράγωνος* (ἥτοι {*ἡγουν* III.C.1 220} ἑκάτερος τῶν ἄκρων) ὁ αὐτὸς μένει [[s.l. 320]] καὶ οὐ μεταβαίνει ἐκ {*ἐκ* s.l. 2450} τοῦ εἶναι τετράγωνος. ὁ δὲ *πρῶτος* ἥτοι ὁ μείζων [[s.l. 320]] *εἰς τὸν ἐλάσσονα μεταβαίνει* [[s.l. 320]], ἥτοι ἀπὸ τοῦ πολλαπλασίου εἶδους εἰς τὸ ἐπιμόριον, καὶ ἀπὸ τοῦ ἐπιμορίου αὐθις εἰς τὸ ἐπιμερές· ἔλαττον γὰρ τοῦ μὲν πολλαπλασίου τὸ ἐπιμόριον, τοῦ δὲ ἐπιμορίου τὸ ἐπιμερές. καὶ ἐξῆς δὲ ἐκάστη σχέσις ὀρθῶς μὲν κειμένη γεννᾷ {*γεννᾶται* III.C.1 220} ἕτερον εἶδος ἐλάττονος σχέσεως, ἀναστρεφομένη δὲ {*δὲ* om. 273} ποιεῖ ἕτερον εἶδος σχέσεως ἐλάττονος καὶ αὐτῆς οὔσης, ὥστε ἐκ μιᾶς σχέσεως διαζεύγνυνται δύο σχέσεις.

b. (marg. inf. 320 : *m. rec. marg. sup. 2450* : *post a* III.C.1 220) γρ. οἱ μὲν ἔσχατοι τετράγωνοι οἱ αὐτοὶ μένουσι, τουτέστι κατὰ τὴν ποιότητα {*ποσότητα* III.C.1} τοῦ τετραγωνισμοῦ, καὶ οὐ κατὰ τὴν ποσότητα τῶν ἐν αὐτοῖς μονάδων.

Adjunct

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοι εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν {s.l. τῶν ἄκρων 320 2450 273 : ἡγουν τῶν ἄκρων 220} ἐπ' ἀλλήλας γενομένων {s.l. πολυπλασιαζομένων, οἷον γ ε, ιε 320 2450 273 220}, καὶ ὁ μὲν πρῶτος {s.l. ὁ μείζων 320 2450 273 220} τῆς ἀπογεννώσεως {s.l. τῆς πολλαπλασίας 320 2450 273 : πολυ– 220} εἰς τὸν ἐλάττονα τῆς γινομένης {s.l. ἡγουν τῆς ἐπιμορίου καὶ ἐξῆς τῆς ἐπιμεροῦς 320 2450 273 220} μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ {s.l. ὁ 320 : οὐ 220 : [[οὐ]] III.C.1} μείζων τετράγωνος ὡσαύτως {ex ὁ αὐτὸς fecit m. rec. 320} ἐστί.

C.15. Glosses and a scholium to the Adjunct by a later hand in Par. gr. 2373, f. 17r

A sign anchors the scholium on the underlined word; the infralinear glosses are also transcribed.

Adjunct

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοι εἰσιν, οἱ δὲ μέσοι {s.l. εἰσὶν} ἐκ τῶν πλευρῶν αὐτῶν ἐπ' ἀλλήλαις γενομένων {s.l. πολλαπλασιασθεισῶν},

καὶ ὁ μὲν πρῶτος {s.l. κατὰ τὴν θέσιν} τῆς ἀπογεννώσης εἰς τὸν ἐλάττονα {s.l. κατὰ τὴν ποσότητα} τῆς γινομένης μεταβαίνει, ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ μείζων τετράγωνος ὁ αὐτός ἐστιν.

οἷον λόγου χάριν ἐν ἀριθμοῖς λόγον ἔχουσιν ἡμιόλιον οἱ δ ς θ. πολλαπλασιάσας γὰρ τὴν πλευρὰν τοῦ δ, ἥτις ἐστὶ πάντως δύο, μετὰ τῆς πλευρᾶς τοῦ θ, ἥτις ἐστὶ γ, ποιήσεις τὸν ς, ὅς ἐστι μέσος· δις γὰρ τρεῖς, ἕξ. καὶ ἐπὶ πασῶν τῶν ἄλλων ἐκθέσεων ὡσαύτως ποιῶν ἄπταιστον εὐρήσεις

C.16. Glosses and scholia in Kharkow, UL, 269-p, 369-c, ff. 87v-88r, and in Par. gr. 2483, pp. 149, 152-153; cf. Laur. Conv. Soppr. 30, f. 43r [a], Oxon. Bodl. Holkham 71, f. 278r [a], Scorial. P.II.3, ff. 433v-434r, 436v-437r [ab]

A sign in 2483 anchors the scholium on the underlined word; the infralinear glosses are also transcribed. Quotations of Ar. I.23.15 are in red.

Ar. I.23.15

ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν {s.l. ἡγουν τῶν ἀντεστραμμένων} καὶ ἀφ' ἧς ἀμφοτέραι {s.l. αἱ ἀντεστραμμέναι καὶ αἱ ὀρθῶς}, ὁ μὲν ἔσχατος τετράγωνος ὦν αὐτὸς μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

C.1 {2483 P.II.3 only}

C.6a + a. <πάντως δὲ οἱ ἄκροι τετράγωνοι.> οἷον ὡς ἐπὶ παραδείγματος {-ατι 30} ἡμιόλιου {-λίοι 30 : ἡμιόλ^λ 71} ἐκκείσθωσαν ὅροι δ ς θ, καὶ ἐξῆς ἐπίτριτοι θ ιβ ις, εἴτα ἐπιτέταρτοι ις κ {κ s.l. 2483 : om. 269-p} κε, καὶ μέχρι {οὗ s.l. m. rec. 2483 : in textu P.II.3} βούλει. ἔχεις οὖν ἐν τῇ ἡμιολίῳ σχέσει πρῶτον τετράγωνον τὸν δ, ὅς {ὅς s.l. m. rec. 2483} οὐκέτι ἐν τῷ ἐπιτρίτῳ {περιτρ- 30 71} φανήσεται, ἀλλ' ἐφ' ἑαυτοῦ μένει· τὸν δὲ θ ἔσχατον αὐτοῦ τετράγωνον ὄντα ὅψει πρῶτον ἐν τῷ ἐπιτρίτῳ. ὁμοίως δὲ καὶ ἐὰν τὸν πρῶτον τετράγωνον τῆς {τὸν 71} ἐπιτρίτου σχέσεως βουλευθῇς ἐν {ἐπὶ 30} τῷ ἐπιτετάρτῳ θεωρῆσαι {-εῖ- 30 71}, οὐκ ὅψει {ἴδε. 30 : εἶδης 71} {, ἀλλ' ἐν τῷ ἐπιτετάρτῳ. P.II.3 : marg. m. rec. 2483 : om. 269-p} ἐὰν δὲ τὸν ἐλάττονα τὸν ις, πρῶτος οὗτος ἐν τῇ ἐπιτετάρτῳ σχέσει φανήσεται σοι {σ. φ. 30 71}, ὅς ἐλάττων ἐστὶ {καὶ 2483 P.II.3} τοῦ ἐπιτρίτου.

To the Adjunct

b. ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων, τουτέστι καὶ τῶν ὀρθῶς καὶ ἀντεστραμμένως κειμένων· ἐκθετέον γὰρ ἀναλογίαν καὶ ὀρθῶς καὶ ἀντεστραμμένως, δ ς θ καὶ ἀντεστραμμένως θ ς δ. φησὶν οὖν ὅτι {ὅτε 2483} οἱ ἄκροι αὐτῶν τετράγωνοι εἰσιν· ὃ τε γὰρ δ καὶ θ τετράγωνοι καὶ ὁ θ καὶ ὁ δ. οἱ δὲ μέσοι αὐτῶν, τουτέστιν οἱ ἕξ, ἐκ τῶν πελυρῶν αὐτῶν τῶν τετραγώνων ἐγένοντο· τοῦ μὲν γὰρ δ πλευρὰ β, τοῦ δὲ θ, τρία, ἕξ

ὧν δύο καὶ τριῶν ἐγένετο ὁ ς μέσος, πλευρῶν ὄντων τοῦ δ καὶ θ τῶν δύο ἄκρων τετραγώνων. πάλιν ὡς ἐξ ἄλλης ἀρχῆς ἢ κειμένη ἀναλογία ἀντεστραμμένως ἢ θ ς δ γεννᾷ τὴν θ ιε κε. **ὁ πρῶτος** οὖν φησὶ τῆς ἀπογεννώσεως {–έσεως P.II.3} σχέσεως **μεταβαίνει εἰς τὸν ἐλάττονα**, ἡγουν ὁ θ γίνεται ἐλάττων μείζων ὧν ἐπὶ τῆς πρώτης. **ἐν ἀμφοτέραις δὲ {ἐλάττων — ἐν bis P.II.3} ταῖς γεννηθείσαις** ἀναλογίαις, δηλονότι ἐκ τε τῆς ὀρθῶς κειμένης καὶ μή, τουτέστι τῆς δ ι κε, θ ιε κε, ὑπάρχει ὁ αὐτὸς καὶ εἰς καὶ ἔσχατος καὶ μείζων, ἡγουν ὁ κε· καὶ ἔσχατος γὰρ κατὰ τὴν {τὴν om. 269-p} τάξιν ἐν ἀμφοτέραις καὶ μείζων ὑπάρχει τῇ ποσότητι {ὁ αὐτὸς καὶ εἰς δηλαδή add. P.II.3 : m. rec. 2483}.

Kharkow, UL, 269-p, 369-c & Par. gr. 2483: sicut novum scholium rursus ἀναλογίαις — ποσότητι ὑπάρχει (τῇ δ ι κε | ὑπάρχει ὁ ἔσχατος καὶ μείζων | κατὰ τὴν τάξιν καὶ ἐν | om. τῇ ποσότητι) sed erasit m. rec. 2483 et post ποσότητι add. ὁ αὐτὸς καὶ εἰς δηλαδή

c. τὰ μὲν λέγεται καὶ ἔστιν, ὡς ὁ ἄνθρωπος· τὰ δὲ οὔτε λέγεται οὔτε ἐστίν, οἷον τὰ μηδαμῇ μηδαμῶς· τὰ δὲ ἔστι μὲν οὐ λέγεται δέ, ὡς τὰ ἐν τῇ Λιβύῃ θηρία· τὰ δὲ λέγεται μὲν οὐκ ἔστι δέ, ὡς εἴ τις εἴποι τὸ τετράγωνον {τρέγωνον 269-p} ἢ τὸ τρίγωνον ἴσον τῷ κύκλῳ κατὰ τὴν χώρησιν, καὶ ἄλλα τοιαῦτα.

C.17. Glosses and scholia in Marc. gr. Z. 316, ff. 60v-61v, Scorial. X.I.9, ff. 46v-47r, 47v-48r, Marc. gr. Z. 319, f. 40r [b]

A sign anchors the scholium on the underlined word; the infralinear glosses are also transcribed.

Ar. I.23.15

ἐπὶ πασῶν δὲ τῶν διαzeugθεισῶν καὶ ἀφ' ἧς ἀμφοτέραι, ὁ μὲν ἔσχατος τετράγωνος {s.l. ἡγουν πρόλογος 316} ὁ αὐτὸς μένει, ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

Marc. gr. Z. 316, f. 60v, Scorial. X.I.9, f. 47r, margin in both manuscripts

a. καὶ γὰρ ἀποδέδεικται γραμμικῶς ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ὥσι πρὸς ἀλλήλους τὸν αὐτὸν λόγον ἔχοντες, οἱ ἄκροι αὐτῶν τετράγωνοι εἰσι κατὰ τὸ πόρισμα τοῦ δευτέρου θεωρημάτος τοῦ η^{ov} τοῦ Εὐκλείδου.

margin in Marc. gr. Z. 316, f. 61r, main text, lesser size script in Scorial. X.I.9, ff. 46v-47r

C.8c

Adjunct

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοι εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν {s.l. τῶν ἄκρων} ἐπ' ἀλλήλαις γενομένων {s.l. ἦτοι πρὸς ἀλλήλας πολυπλασιασθεισῶν}, καὶ ὁ μὲν πρῶτος {s.l. κατὰ τὴν θέσιν} τῆς ἀπογεννώσεως {s.l. τῆς διπλασίας φέρε εἰπεῖν ἐντεστραμμένης ἢ καὶ ὀρθῶς κειμένης : –ω –ω X.I.9} εἰς

τὸν ἐλάττονα τῆς γεννωμένης {s.l. σχέσεως οἶον τῆς ἡμιολίου} μεταβαίνει, ἐν ἀμφοτέ-
ραις δὲ ταῖς γεννηθείσαις ὁ ἔσχατος {s.l. τῇ θέσει} καὶ μείζων τετράγωνος {s.l. τῷ λόγῳ,
πρόλογος ὢν} ὁ αὐτός {s.l. ἡγουν πρόλογος} ἐστίν.

after the end of Book I, lesser size script in Marc. gr. Z. 316, f. 61v; main text, lesser size script in
Scorial. X.I.9, ff. 47v-48r; main text, after Texts **C.8bc** in Marc. gr. Z. 319, f. 40r

b. παραδειγματιστέον ἐπὶ τῶν διπλασίων πῶς οἱ μέσοι γεννῶνται ἐκ τῶν πλευρῶν τῶν
ἄκρων πρὸς ἀλλήλας πολυπλασιασθεισῶν. ἔστωσαν γὰρ α β δ. ὁ γοῦν μέσος (ἡγουν ὁ β)
γεννᾶται ἐκ τῶν πλευρῶν τῶν ἄκρων (ἦτοι τοῦ α καὶ τοῦ β {δ 319}) πρὸς ἀλλήλας πολυ-
πλασιασθεισῶν· πλευρὰ γὰρ τῆς μὲν μονάδος ἢ μονάς, τῆς δὲ τετράδος {δ 316} ἢ δυάς
{τετράς 319}. πολυπλασιασθεισῶν γοῦν τούτων {πολλαπλασιασθείσας γοῦν ταύτας 319}
πρὸς ἀλλήλας γεννᾶται ὁ μέσος· ἅπαξ γὰρ τὰ δύο, δύο. καὶ ἐπὶ τῶν λοιπῶν ὡσαύτως.
ὅρα δὲ καὶ ἐπὶ ἐτέρων ἐκθέσεων {-εως 319} τὸ λεγόμενον.

**C.18. Glosses and a scholium in Laur. Conv. Soppr. 30, ff. 42v-43r, and Oxon. Bodl.
Holkham 71, f. 278r-v [only the scholia]**

Ar. I.23.15

ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν {s.l. ἐπιμορίων καὶ ἐπιμερῶν} καὶ ἀφ' ἧς ἀμφοτέραι {s.l.
τῆς πολυπλασίῳ}, ὁ μὲν ἔσχατος {s.l. ἄκρος} τετράγωνος ὁ αὐτὸς μένει, ὁ δὲ πρῶτος {s.l.
μείζων λόγος} εἰς τὸν ἐλάττονα {s.l. λόγον} μεταβαίνει, πάντως δὲ οἱ ἄκροι τετράγωνοι.

C.6a + C.16a

ὁ ρκς ἔχει τὸν πα καὶ ἑαυτοῦ ἔννατον, ἦτοι με. ὁ δὲ ρος ἔχει τὸν ρκς. πεντάκις δὲ ιδ, ο. ἀπὸ
δὲ τοῦ ἐλάττονος κε ὁ ρος διπλασιεπιτετράπεμτοι· ὁ γὰρ ο τὸν κε ἔχει δις καὶ τέσσαρα
αὐτοῦ πέμπτα, καὶ ὁ ρος τὸν ο δις καὶ δ αὐτοῦ πέμπτα· παντάκις γὰρ ὁ ιδ, ο· καὶ τεσσα-
ράκις {τετράκις 71} ιδ, νς, ὡς δις τὰ ο καὶ νς, ρος. ἰστέον ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι
ᾧσι {ὡς οἱ 71} πρὸς ἀλλήλους τὸν αὐτὸν λόγον ἔχοντες, πάντως οἱ ἄκροι αὐτῶν τετρά-
γωνοὶ εἰσιν, ἐλάχιστοι δὲ οὔτοι οἱ προκείμενοι λέγονται· διότι δύο μὲν εὐρήσεις ὑποκάτω
αὐτῶν πυθμένας, τρεῖς δὲ οὔ.

Adjunct

ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων πάντως οἱ ἄκροι τετράγωνοι εἰσιν, οἱ δὲ μέ-
σοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ' ἀλλήλαις γενομένων, καὶ ὁ μὲν πρῶτος τῆς ἀπογεννώσεως
{s.l. τῆς πολυπλασίῳ} εἰς τὸν ἐλάττονα {s.l. λόγον} τῆς γινομένης μεταβαίνει, {add. καὶ}
ἐν ἀμφοτέραις {s.l. ἐπιμορίοις καὶ ἐπιμερέσι} δὲ ταῖς γεννηθείσαις ὁ ἔσχατος καὶ μείζων
τετράγωνος ἐστίν.

6. Philological Remarks

6.1. Nicomachus' *Introductio arithmetica*

In the previous Section I have collected the textual data related to two sentences of *Ar.*: section I.23.15 and an adjunct that is attached to the end of Book I in most manuscripts and that Hoche prints in his apparatus. The Adjunct is nothing but a paraphrase, in reverse order, of *Ar.* I.23.15. The following table compares Version A of I.23.15 and Version C.5.1.A of the Adjunct, the latter suitably reassembled (the numbers restore the order).

<i>Ar.</i> I.23.15	Adjunct	
ἐπὶ πασῶν δὲ τῶν διαζευχθεισῶν	ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων	1
καὶ ἀφ' ἧς ἀμφοτέραι,	ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις	4
ὁ μὲν ἔσχατος τετράγωνος ὁ αὐτὸς μένει,	ὁ ἔσχατος καὶ μείζων τετράγωνος ὁ αὐτὸς ἐστίν	5
ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει,	καὶ ὁ μὲν πρῶτος τῆς ἀπογεννώσεως εἰς τὸν ἐλάττονα τῆς γινομένης μεταβαίνει,	3
πάντως δὲ οἱ ἄκροι τετράγωνοι	πάντως οἱ ἄκροι τετράγωνοι εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ' ἀλλήλαις γενομένων,	2

As said, *Ar.* I.23.15 is unanimously witnessed to, whereas fifteen manuscripts out of eighty-four do not have the Adjunct. Among them, we find the two earliest witnesses of *Ar.*: Matrit. 4678 and Par. gr. 2479. In the latter, the Adjunct is added by a hand of end 13th – beginning 14th century (see Section 6.3 below). These facts appear to exclude the possibility that the Adjunct is part of the original *Ar.*, even if a wild plot featuring a wholesale and very early omission of the Adjunct in the exemplars read by Iamblichus and Ammonius (who do not read it at the end of their *Ar.*)³³ and its surfacing again in late 13th-century copies cannot be ruled out as utterly unreal. In this scenario, *Ar.* I.23.15 is a clumsy attempt at restoring, in the wrong place, the original, and much clearer, final sentence. The Adjunct is in fact the clearest yet concise interpretation of I.23.15 in the entire exegetical record;³⁴ its real nature of adjunct is made plain by the second clause in sentence (2) above, a well-known property that, however, has no bearing on the context of I.23.15.

³³ Maybe I should be less categorical: after all, our Text C.3 is the last scholium of Rec. III, and the scholium before it comments on *Ar.* I.23.11. Thus, nothing can be said about the position of the clause explained in Text C.3. Note, moreover, that C.3 only comments on one of the three claims in I.23.15. On these grounds, the reader is urged to conceive a scenario even wilder than the one I have just sketched.

³⁴ See Section 7 below. There remains just one designation quirk in the Adjunct.

From the philological point of view, *Ar.* I.23.15 and the Adjunct raise important methodological problems. First and foremost, if I.23.15 is an interpolation, then the manuscript tradition of *Ar.* admits of an archetype, defined (as it should) by the set of non-original readings shared by the entire tradition: this set may well be a singleton.³⁵ Of course, the existence of an archetype presupposes adherence to the genealogical paradigm; if contamination takes place, this cannot be taken for granted. Well, the Adjunct shows what “contamination” might have meant in our case. For in Vat. gr. 2297 + Roma, Arch. S. Paolo 24C, an important witness of the late 13th century,³⁶ the Adjunct lies in the margin of Rec. I, which in its turn lies in the margins of *Ar.*; in its apograph Gott. philol. 66, the Adjunct is relocated at the end of Book I of Rec. I, but a later hand copies it again, preceded by κείμενον, in a blank space in the same page and in the subsequent, blank, page. Well, in Par. gr. 2480, a slavish copy of the *Gottingensis*, we read the Adjunct both at the end of Rec. I and at the end of *Ar.*, whereas in another conformal copy of it, namely, Bucur., BAR gr. 620, the Adjunct can only be read as a last sentence of *Ar.*³⁷ On the other hand, in manuscripts such as Par. gr. 2107, the Adjunct is marked for deletion by a later hand, and might well have been deleted in later copies.

The bottom line of all of this is that the Adjunct may have (or actually has) entered or left the text of *Ar.* by contamination with the exegetical apparatus and by collation initiatives of copyists or scholars. Of course, the same might well have happened in the case of *Ar.* I.23.15: imagine two, or even more, “real” but lost witnesses that served as roots of the tradition, and such that only some of them have I.23.15 in the text, whereas the others either have it in the margin or do not have it at all, but in all of their copies I.23.15 has been included in *Ar.*, possibly by contamination with the exegetical apparatus or with the exemplars that do have I.23.15. Yet, if this *might* have occurred, then it *might* also *have not*. And if we want to keep within the stemmatic paradigm, these

³⁵ On the meaning of “archetype” and “contamination”, see, most recently, the excellent discussion in P. Trovato, *Everything You Always Wanted to Know about Lachmann's Method. A Non-Standard Handbook of Genealogical Textual Criticism in the Age of Post-Structuralism, Cladistics, and Copy-Text*, *Storie e linguaggi* 7, Padova 2017², sect. 3, *passim*, with bibliography; and also Ph. Roelli (ed.), *Handbook of Stemmatics. History, Methodology, Digital Approaches*, Berlin 2020, sects. 2 and 4.1.

³⁶ See C. Sojer, “Un codice dimenticato di Nicomaco di Gerasa, riscritto su testi innografici e sul *Commento a Giovanni* di Cirillo di Alessandria”, *Nέα Πώμη* 8 (2011), 199-217; and C. Hofstetter, “Un nouveau témoin complet de l'*Introduction arithmétique* de Nicomaque de Gerasa”, *Nέα Πώμη* 15 (2018), 177-192, on this manuscript.

³⁷ For these manuscripts, check I.23.15, Version A, and Texts **C.1**, **C.5.1A** and **C.11a**, and cf. Section 6.3 below.

possible-world scenarios are excluded by definition: if there is an innovation shared by the entire tradition, then the stemma has one single root, called “archetype”. All of this shows, a fact that is frequently forgotten, that stemmatics is a formal method grounded on a strict principle of parsimony.³⁸ Since *any* kind of innovation can be supposed to be transmitted by contamination in a suitably wild scenario, the alternative to stemmatics is in fact no method at all –even in the case of *Ar.*, whose *varia lectio* displays a massive contamination even at pre-traditional stages. The fact that *some* wild scenarios actually occurred does not mean that *any* wild scenario has: counterexamples are a warning, not a proof.

Both *Ar.* I.23.15 and the Adjunct display several distinctive variant readings, which realize a partition of the textual tradition of *Ar.*; some of these subsets of manuscript witnesses are very small (see Versions **CE** of I.23.15 and **C.5.1D** of the Adjunct), some obviously characterize the revised text of *Ar.* attached to Rec. II (Version **D**, with some manuscripts entering the group by polygenesis, and **C.5.1E**). Some of the variants are or include sheer mistakes (Versions **BCE** and **C.5.1CDF**); some offer a very bewildering text (**C.5.1D**); some are inherited and some have an obvious polygenetic nature. But many of the variant readings perfectly fit the syntactical and mathematical context: this shows how difficult is to apply the genealogical method in the topmost portion of the stemma of *Ar.* The most important such variant (**D** and **C.5.1B**) has πάντες instead of πάντως in both I.23.15 and the Adjunct, a reading that is also witnessed to in one of the running commentaries on *Ar.* To them we now turn our attention.

6.2. The “Recensions” of Ammonius’ lecture notes

I first set out our extracts of the four “Recensions” in three comparative tables; these are Texts **C.1** to **C.4** above. The first two tables account for the evolution Rec. III → Rec. I → Rec. II; the third table compares Rec. I and Rec. IV (in this case, similar yet differently formulated argumental items are in red, an important variant reading is in blue). Colour markings highlight the changes from Rec. III to Rec. I (modified items: orange; items absent in the other Recension: blue) and from Rec. I to Rec. II (different items: green; an important variant reading is in blue). Textual units corresponding to each other are located in facing cells. Take in due account the fact that the text of Rec. I and II is not a critical text.

³⁸ Thus, the archetype is a formal object, which need not coincide with a manuscript. But if it did, such a manuscript must be non-extant.

Rec. III, sect. I.ρνζ	Rec. I, sect. I.ρπς
	διαζευχθεῖσαι εἰσι σχέσεις αἱ τε κατ' ὀρθὸν ἐπιζευγνύμεναι καὶ αἱ κατὰ ἀναστροφὴν συμπλεκόμεναι· διέζευκται γὰρ τοῦ ὀρθοῦ τὸ ἀνεστραμμένον· ἐκάστης μὲν οὖν σχέσεως οἱ ἄκροι πάντως εἰσὶ τετράγωνοι, δι' ἣν αἰτίαν ἐροῦμεν·
	ἀλλ' ὁ μὲν ἔσχατος ὁ αὐτὸς μένει, φησὶν· οὐ πρόεισι γὰρ ἐφ' ἕτερον ἢ μειούμενος ἢ αὐξάμενος, ἔσχατος ὢν· ὁ δὲ πρῶτος ἐκ τῆς ἀναστροφῆς πρὸς τὸν ἐλάττονα πρόεισι μέχρι μονάδος, ἐπεὶ καὶ ἀπὸ μονάδος ἤρξατο. τοῦτο δὲ δηλὸν ἔκ τε τῶν προειληφνυῶν σχέσεων καὶ τῶν λέγεσθαι μελλουσῶν.
οἱ γὰρ ἄκροι πάντως τετράγωνοι εὐρεθήσονται, ὡς ἐπὶ μὲν τῶν ἡμιολίων τοῦ δ καὶ τοῦ ς καὶ τοῦ θ, ὁ δ καὶ ὁ θ· ἐπὶ δὲ τῶν ἐπιτρίτων τοῦ θ καὶ τοῦ ιβ καὶ τοῦ ις, ὁ θ καὶ ὁ ις· καὶ ἐπὶ πάντων τῶν εἰδῶν τετράγωνοι εἰσιν οἱ ἄκροι, καὶ τοῦτο εὐλόγως,	πάντως δέ, φησὶν, οἱ ἄκροι τετράγωνοι· ἐπὶ μὲν ἡμιολίων, τοῦ δ καὶ ς καὶ θ, ὁ δ καὶ ὁ θ· ἐπὶ δὲ τῶν ἐπιτρίτων, τοῦ θ καὶ ιβ καὶ ις, ὁ θ καὶ ὁ ις, καὶ ἐπὶ πάντων ὁμοίως· καὶ τοῦτο εὐλόγως·
ἐπειδὴ δέδεικται γραμμικῶς ὅτι ἂν τρεῖς ἀριθμοὶ ἐλάχιστοι ὣσι πρὸς ἀλλήλους τὸν αὐτὸν λόγον ἔχοντες, οἱ ἄκροι αὐτῶν τετράγωνοι εἰσιν. ἐπεὶ οὖν καὶ ἐπὶ τῶν ἡμιολίων ὁ δ καὶ ὁ ς καὶ ὁ θ ἐλάχιστοι εἰσι, διὰ τοῦτο οἱ ἄκροι τετράγωνοι εἰσιν.	ἐδείχθη γὰρ παρὰ τῷ γεωμέτρῃ ὅτι ἂν τρεῖς ἀριθμοὶ ἐλάχιστοι ὣσι πρὸς ἀλλήλους τὸν αὐτὸν λόγον ἔχοντες, οἱ ἄκροι αὐτῶν τετράγωνοι εἰσιν. ἐπεὶ οὖν ἐπὶ τῶν ἡμιολίων ὁ δ καὶ ὁ ς καὶ ὁ θ ἐλάχιστοι πρὸς ἀλλήλους εἰσὶ καὶ τὸν αὐτὸν ἔχουσι λόγον, διὰ τοῦτο οἱ ἄκροι τούτων τετράγωνοι εἰσιν.
ἀλλ' ἴσως εἴποι τις ὅτι οὐκ εἰσὶν ἐλάχιστοι, οὐδὲ γὰρ πυθμένες εἰσὶ τῶν ἡμιολίων· ἰδοὺ γάρ, ὁ γ τοῦ β ἡμιόλιος.	ἀλλ' ἴσως εἴποι τις ἄν· οὐκ εἰσὶν ἐλάχιστοι· οὐδὲ γὰρ εἰσι πυθμένες τῶν ἡμιολίων· ἰδοὺ γάρ, ὁ γ τοῦ β ἡμιόλιός ἐστιν.
φαμὲν ὅτι δύο μὲν ἀριθμοὶ ἡμιόλιοι ἐλάχιστοι εὐρίσκονται, ὡς ὁ β καὶ ὁ γ, τρεῖς δὲ οὐκέτι· οὐκ ἂν γὰρ ὑποκάτω τοῦ δ καὶ τοῦ ς καὶ τοῦ θ τρεῖς ἡμιολίους εὕρῃς. ὡσαύτως δὲ καὶ δύο μὲν ἐπιτρίτους εὐρίσκεις, τὸν γ καὶ τὸν δ· τρεῖς δὲ ὑποκάτω τοῦ θ καὶ τοῦ ιβ καὶ τοῦ ις οὐκ ἂν εὔροις.	πρὸς ὃ λέγομεν ὅτι δύο μὲν ἐφεξῆς ἔσχατοι ἡμιόλιοι εὐρίσκονται, ὡς τὰ β καὶ τὰ γ, τρεῖς δὲ ἐφεξῆς ἐλάττονες τοῦ δ καὶ ς καὶ τοῦ θ οὐδέποτε. ὡσαύτως καὶ δύο μὲν ἐπιτρίτους, τὸν γ καὶ τὸν δ, εὐρεῖν ἔστι, τρεῖς δὲ ἐφεξῆς ὑποκάτω τοῦ θ καὶ τοῦ ιβ καὶ τοῦ ις ἀδύνατον.
καὶ ἐπὶ πάντων τῶν ἄλλων ὁμοίως, ὥστε εἰκότως τοὺς ἄκρους τετραγώνους ἔχουσιν.	καὶ ἐπὶ τῶν ἄλλων ὡσαύτως· εἰκότως οὖν οὗτοι οὖν τοὺς ἄκρους τετραγώνους ἔχουσιν.

Rec. I, sect. I.ρπς	Rec. II, sect. I.<ρ>νζ
διαzeugθεῖσαι εἰσι σχέσεις αἱ τε κατ' ὀρθὸν ἐπιzeugνύμεναι καὶ αἱ κατὰ ἀναστροφὴν συμπλεκόμεναι· διέzeugται γὰρ τοῦ ὀρθοῦ τὸ ἀνεστραμμένον· ἐκάστης μὲν οὖν σχέσεως οἱ ἄκροι πάντως εἰσὶ τετράγωνοι, δι' ἣν αἰτίαν ἐροῦμεν· ἀλλ' ὁ μὲν ἔσχατος ὁ αὐτὸς μένει, φησὶν· οὐ πρόεισι γὰρ ἐφ' ἕτερον ἢ μειούμενος ἢ αὐξάμενος, ἔσχατος ὢν· ὁ δὲ πρῶτος ἐκ τῆς ἀναστροφῆς πρὸς τὸν ἐλάττονα πρόεισι μέχρι μονάδος, ἐπεὶ καὶ ἀπὸ μονάδος ἤρξατο. τοῦτο δὲ δηλὸν ἐκ τῶν προειληφειῶν σχέσεων καὶ τῶν λέγεσθαι μελλουσῶν.	διαzeugθεῖσαι σχέσεις εἰσὶν αἱ τε κατ' ὀρθότητα τῶν προτέρων ἐκκειμένων ἀπογεννώμεναι καὶ αἱ κατὰ ἀναστροφὴν· διέzeugται γὰρ τοῦ ὀρθοῦ τὸ ἀνεστραμμένον· ἐφ' ἐκάστης οὖν φησι σχέσεως οἱ ἄκροι πάντως εἰσὶ τετράγωνοι, δι' ἣν αἰτίαν ἐροῦμεν· ἀλλ' ὁ μὲν ἔσχατος, τουτέστιν ὁ ὑπόλογος ἐν πάσαις ταῖς ἐκθέσεσι ταῖς τε κατ' ὀρθότητα ἐκκειμέναις ταῖς τε κατ' ἀντιστροφὴν, τετράγωνος ὢν ὁ αὐτὸς μένει, τουτέστιν ὑπόλογος, οἷον ἐν ἡμιολίῳ σχέσει ὀρθῶς ἐκκειμένη ἦσαν ὁ δ ς θ, καὶ πάντως ὑπόλογος ἦν ἐν τούτοις ὁ δ (τοῦτον γὰρ φησιν ἔσχατον)· ἀντιστραφέντες δ' ἦσαν οὕτως θ ς δ, καὶ πάλιν ὁ δ, εἰ καὶ ὕστερος τέτακται, ἀλλὰ πάλιν ὑπόλογός ἐστι πρὸς τὸν ς· ὁ δὲ πρῶτος, ἡγουν ὁ μέγιστος πρόλογος, εἰ ἀπὸ τούτου ἄρχοιτο δηλαδὴ ἡ τῆς ἐξῆς σχέσεως πρόβασις, ὑπόλογος γίνεται ἐν ἐκείνῃ. οἷον ἔστω ἡ ῥηθεῖσα ἡμιόλιος σχέσις ἀντεστραμμένη, ἡγουν θ ς δ· ἐκ ταύτης γίνεται ἄλλη σχέσις θ ιε κε. φανερόν οὖν ὅτι ἐν μὲν ἐκείνοις πρόλογος ἦν ὁ θ καὶ μέγιστος, ἐν δὲ τούτοις καὶ ὑπόλογος καὶ ἐλάχιστος.
πάντως δέ, φησὶν, οἱ ἄκροι τετράγωνοι· ἐπὶ μὲν ἡμιολίων, τοῦ δ καὶ ς καὶ θ, ὁ δ καὶ ὁ θ· ἐπὶ δὲ τῶν ἐπιτρίτων, τοῦ θ καὶ ιβ καὶ ις, ὁ θ καὶ ὁ ις, καὶ ἐπὶ πάντων ὁμοίως· καὶ τοῦτο εὐλόγως· ἐδείχθη γὰρ παρὰ τῷ γεωμέτρῃ ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ὡς πρὸς ἀλλήλους τὸν αὐτὸν λόγον ἔχοντες, οἱ ἄκροι αὐτῶν τετράγωνοι εἰσιν. ἐπεὶ οὖν ἐπὶ τῶν ἡμιολίων ὁ δ καὶ ὁ ς καὶ ὁ θ ἐλάχιστοι πρὸς ἀλλήλους εἰσὶ καὶ τὸν αὐτὸν ἔχουσι λόγον, διὰ τοῦτο οἱ ἄκροι τούτων τετράγωνοι εἰσιν.	πάντες δέ, φησὶν, οἱ ἄκροι τετράγωνοι· ἐπὶ μὲν γὰρ τῶν ἡμιολίων, τοῦ δ καὶ ς καὶ θ, ὁ δ καὶ ὁ θ τετράγωνοι· ἐπὶ δὲ τῶν ἐπιτρίτων, τοῦ θ καὶ ιβ καὶ ις, ὁ θ καὶ ὁ ις, καὶ ἐπὶ τῶν ἄλλων ὁμοίως· καὶ τοῦτο εἰκότως· ἐδείχθη γὰρ παρὰ τῷ γεωμέτρῃ ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ὡς πρὸς τὸν αὐτὸν λόγον ἔχοντων αὐτοῖς, οἱ ἄκροι αὐτῶν τετράγωνοι εἰσιν. ἐπεὶ οὖν ἐπὶ τῶν ἡμιολίων ὁ δ, ὁ ς καὶ ὁ θ ἐλάχιστοί εἰσι πρὸς ἀλλήλους τῶν τὸν αὐτὸν λόγον ἔχοντων αὐτοῖς (οὐδὲ γὰρ ἔστιν εὐρεῖν πρὸ αὐτῶν ἡμιολίους τρεῖς ἐφεξῆς), διὰ τοῦτο οἱ ἄκροι αὐτῶν τετράγωνοι εἰσι.
ἀλλ' ἴσως εἴποι τις ἄν· οὐκ εἰσὶν ἐλάχιστοι· οὐδὲ γὰρ εἰσι πυθμένες τῶν ἡμιολίων· ἰδοῦ γάρ, ὁ γ τοῦ β ἡμιολίός ἐστιν.	ἀλλ' ἴσως εἴποι τις ἄν τὸν δ καὶ ς καὶ θ μὴ εἶναι ἐλάχιστους ἐν ἡμιολίῳ λόγῳ· οὐδὲ γὰρ εἰσι πυθμένες τῶν ἡμιολίων· ὁ β γὰρ καὶ ὁ γ εἰσὶ πυθμένες τοῦ ἡμιολίου.
πρὸς δ λέγομεν ὅτι δύο μὲν ἐφεξῆς ἔσχατοι ἡμιόλιοι εὐρίσκονται, ὡς τὰ β καὶ τὰ γ, τρεῖς δὲ ἐφεξῆς ἐλάττονες τοῦ δ καὶ ς καὶ τοῦ θ οὐδέποτε. ὡσαύτως καὶ δύο μὲν ἐπιτρίτους, τὸν γ καὶ τὸν δ, εὐρεῖν ἔστι, τρεῖς δὲ ἐφεξῆς ὑποκάτω τοῦ θ καὶ τοῦ ιβ καὶ τοῦ ις ἀδύνατον.	πρὸς δ λέγομεν ὅτι δύο μὲν ἐφεξῆς ἡμιόλιοι ἔσχατοι καὶ πυθμένες εἰσὶν ὁ β καὶ ὁ γ, τρεῖς δὲ ἐφεξῆς ἐλάττονες τοῦ δ καὶ ς καὶ θ οὐκ εἰσιν. ὡσαύτως δὲ καὶ δύο μὲν ἐπιτρίτους ἐλάχιστους ἔστιν εὐρεῖν τὸν γ καὶ τὸν δ, τρεῖς δὲ ἐλάσσονας τῶν θ καὶ ιβ καὶ ις οὐδαμῶς.
καὶ ἐπὶ τῶν ἄλλων ὡσαύτως· εἰκότως οὖν οὔτοι οὐ τοὺς ἄκρους τετραγώνους ἔχουσιν.	καὶ ἐπὶ τῶν ἄλλων ὡσαύτως· εἰκότως οὖν οὔτοι τοὺς ἄκρους τετραγώνους ἔχουσιν.

Rec. I, sect. I, ρς	Rec. IV
διαzeugθεισάι εἰσι σχέσεις αἱ τε κατ' ὀρθὸν ἐπιzeugνύμεναι καὶ αἱ κατὰ ἀναστροφὴν συμπλεκόμεναι· διέzeugται γὰρ τοῦ ὀρθοῦ τὸ ἀνεστραμμένον· ἐκάστης μὲν οὖν σχέσεως οἱ ἄκροι πάντως εἰσὶ τετράγωνοι, δι' ἣν αἰτίαν ἐροῦμεν·	ἐπὶ πάσης τῶν διαzeugθεισῶν ἥτοι ἐπὶ πάσης τῆς ὀρθῆς καὶ ἀντεστραμμένης λήψεως – διέzeugται γὰρ ἢ κατ' εὐθεῖαν καὶ κατὰ ἀντιστροφὴν λήψις – ἀφ' ἧς τῆς ὀρθῆς καὶ ἀντεστραμμένης λήψεως ἀμφοτέραι γεννῶνται αἱ σχέσεις· εἰ μὲν κατ' εὐθεῖαν ληφθῇ, γεννᾶται ἡ πολλαπλασιεπιμόριος σχέσις· εἰ δὲ ἀντεστραμμένως, ἡ ἐπιμερής·
ἀλλ' ὁ μὲν ἔσχατος ὁ αὐτὸς μένει, φησὶν· οὐ πρόεισι γὰρ ἐφ' ἕτερον ἢ μειούμενος ἢ αὐξάμενος, ἔσχατος ὢν· ὁ δὲ πρῶτος ἐκ τῆς ἀναστροφῆς πρὸς τὸν ἐλάττονα πρόεισι μέχρι μονάδος, ἐπεὶ καὶ ἀπὸ μονάδος ἤρξατο· τοῦτο δὲ δῆλον ἐκ τε τῶν προειληφειῶν σχέσεων καὶ τῶν λέγεσθαι μελλουσῶν·	ὁ μὲν ἔσχατος τετράγωνος, ἥτοι ὁ δ· ἔσχατος δὲ ἐστὶ διότι, εἰ ἀντιστρέφεις καὶ λάβῃς τὸν θ πρῶτον, ἔσχατος καταλιμπάνεται ὁ δ· ἐπὶ τοῦ ἡμιολίου, οἷον εἰ εἶπω ἴσον πρῶτῳ θέσθαι καὶ ἐκθήσω τὸν θ, εἴτα ἴσον πρῶτῳ καὶ δευτέρῳ καὶ ἐκθήσω τὸν ιε, ἔσχατος κατελείφθη ὁ δ τετράγωνος· οὗτος οὖν ὁ αὐτὸς μένει, καὶ οὐ γίνεται ὁ δ ἀρχὴ καὶ κορυφὴ ἐλάττονός τινος ἀριθμοῦ ἢ τοῦ ἐπιτρίτου ἢ τοῦ ἐπιτετάρτου, ἀλλ' ὁ αὐτὸς μένει καὶ οὐ μεταβαίνει ἀπὸ τοῦ ἡμιολίου εἰς ἕτερον ἐλάττονα· ὁ δὲ θ τετράγωνος ὢν καὶ αὐτὸς πρῶτος δὲ ἀντεστραμμένως μεταβαίνει εἰς τὸν ἐλάττονα (ἥτοι τὸν ἐπίτρίτον)· γίνεται γὰρ κορυφὴ καὶ ἀρχὴ τοῦ ἐπιτρίτου θ ις, ὁ δὲ ἐπίτρίτος ἐλάττων ἐστὶ τοῦ ἡμιολίου· πᾶν γὰρ μέγεθος εἰς δύο μείζονα τέμνεται τὰ ἡμίση· τέμνεται δὲ εἰς ἐλάττονα, εἰ διαιρεθῇ εἰς τρία μέρη· καὶ πάλιν ὁ θ ἐν τῷ ἐπιτρίτῳ ἔσχατος ὢν ἀντεστραμμένως οὐ μεταβαίνει εἰς ἐλάττονα ἀλλὰ μένει ὁ αὐτός· ὁ δὲ ις τετράγωνος πρῶτος ὢν κατ' ἀντιστροφὴν – εἰ γὰρ ἀντιστρέψεις καὶ εἴπῃς ἴσον πρῶτῳ θέσθαι καὶ ἐκθήσεις τὸν ις, ἔστιν ὁ ις ἀντεστραμμένως πρῶτος – ὁ δὲ θ ἔσχατος· ὁ οὖν ις μεταβαίνει εἰς τὸν ἐλάττονα τοῦ ἐπιτετάρτου καὶ ἀρχὴ γίνεται αὐτοῦ· ις κ κε· ὁ δὲ ἐπιτετάρτος ἐλάττων ἐστὶ τοῦ ἐπιτρίτου καθ' ὃν τρόπον καὶ ὁ ἐπίτρίτος τοῦ ἡμιολίου· οὗτος δὲ ὁ λόγος καὶ ἡ μετάβασις τοῦ πρῶτου κατ' ἀντιστροφὴν εἰς τὸν ἐλάττονα ἐπὶ μόνων τῶν ἐπιμορίων σφίξει·
πάντως δέ, φησὶν, οἱ ἄκροι τετράγωνοι· ἐπὶ μὲν ἡμιολίων, τοῦ δ καὶ ς καὶ θ, ὁ δ καὶ ὁ θ· ἐπὶ δὲ τῶν ἐπιτρίτων, τοῦ θ καὶ ιβ καὶ ις, ὁ θ καὶ ὁ ις, καὶ ἐπὶ πάντων ὁμοίως· καὶ τοῦτο εὐλόγως·	πάντες δὲ οἱ ἄκροι· δ ς θ· καὶ ὁ δ καὶ ὁ θ τετράγωνοι· θ ιβ ις· καὶ ὁ θ καὶ ὁ ις τετράγωνοι· δ ι κε· καὶ ὁ δ καὶ ὁ κε τετράγωνοι, καὶ ἀπλῶς πάντες οἱ ἄκροι τετράγωνοι·
ἐδείχθη γὰρ παρὰ τῷ γεωμέτρῳ ὅτι ἐὰν τρεῖς ἀριθμοὶ ἐλάχιστοι ὦσι πρὸς ἀλλήλους τὸν αὐτὸν λόγον ἔχοντες, οἱ ἄκροι αὐτῶν τετράγωνοί εἰσιν· ἐπεὶ οὖν ἐπὶ τῶν ἡμιολίων ὁ δ καὶ ὁ ς καὶ ὁ θ ἐλάχιστοι πρὸς ἀλλήλους εἰσὶ καὶ τὸν αὐτὸν ἔχουσι λόγον, διὰ τοῦτο οἱ ἄκροι τούτων τετράγωνοί εἰσιν· ἀλλ' ἴσως εἴποι τις ἄν· οὐκ εἰσὶν ἐλάχιστοι· οὐδὲ γὰρ εἰσι πυθμένες τῶν ἡμιολίων· ἰδοὺ γάρ, ὁ γ τοῦ β ἡμιολιός ἐστιν· πρὸς δ λέγομεν ὅτι δύο μὲν ἐφεξῆς ἔσχατοι ἡμιόλιοι εὐρίσκονται, ὡς τὰ β καὶ τὰ γ, τρεῖς δὲ ἐφεξῆς ἐλάττονες τοῦ δ καὶ ς καὶ τοῦ θ οὐδέποτε· ὡσαύτως καὶ δύο μὲν ἐπιτρίτους, τὸν γ καὶ τὸν δ, εὐρεῖν ἔστι, τρεῖς δὲ ἐφεξῆς ὑποκάτω τοῦ θ καὶ τοῦ ιβ καὶ τοῦ ις ἀδύνατον· καὶ ἐπὶ τῶν ἄλλων ὡσαύτως· εἰκότως οὖν οὔτοι οὐ τοὺς ἄκρους τετραγώνους ἔχουσιν·	δέδεικται γὰρ ἐν γεωμετρίᾳ ὅτι ὅταν τρεῖς ἀριθμοὶ ἴσην ἀναλογίαν ἔχωσι πρὸς ἀλλήλους (οἷον δ ς θ· ὃν γὰρ λόγον ἔχει ὁ ς πρὸς τὸν δ, τοῦτον ἔχει ὁ θ πρὸς τὸν ς), τούτων οἱ ἄκροι εἰσὶ τετράγωνοι·

The first two tables display a typical pattern for the evolution Rec. III → Rec. I → Rec. II: local rewritings and growth by inflation. That this was the case in the transition from Rec. III to Rec. I has already been shown by Westerink and Tarán, who also surmised –rightly in my opinion– that Rec. III is more faithful to Ammonius’ lecture notes than Rec. I, and that the latter might even have used the former as its sole source.³⁹ In our case, Philoponus adds a (wrong) clarification of the part of *Ar.* I.23.15 left uncommented by Rec. III (second cell in the table), smooths out the syntax and regularizes the lexicon, and unduly (but not unreasonably given the context) adds a crucial negation in the final sentence.⁴⁰

The second table shows, a fact that is confirmed by a complete collation of the text as I have undertaken, that Rec. II is a recension of Rec. I according to the same principles as are used by Philoponus to modify Asclepius’ text: local rewritings and growth by inflation. In particular, Rec. II radically rewrites Rec. I’s wrong clarification (second cell), adds some specific explanations, makes some expressions more explicit, deletes the final negation, aligns the quote from *Ar.* with its own text of *Ar.*,⁴¹ changes the pseudo-citation of the *Elements* (where no theorem with such an enunciation exists) to a wording more faithful to a formulation consistently adopted in the Euclidean treatise.⁴² There can be no doubt, in my opinion, that Rec. II is a Byzantine recension of Rec. I.⁴³ Tannery surmised that the reviser was Isaak Argyros,⁴⁴ to whom some scholia are expressly ascribed in the margins of Vat. gr. 1411,⁴⁵ which, as far as my documentary record goes, is in fact the only independent witness of Rec. II.⁴⁶ Tannery’s hypothesis must be false exactly for the reason he adduces: were Argyros the reviser, he would not have kept his own name by

³⁹ Westerink, “Deux commentaires” (cit. n. 19), and Tarán, *Asclepius* (cit. n. 19), 8-17.

⁴⁰ Several late witnesses perceived the problem and deleted this negation: check Text **C.1**.

⁴¹ Qualifier πάντες instead of πάντως, at the beginning of the third cell: see I.23.15, Version **D**. The same reading is adopted by Rec. IV.

⁴² The designation “among the numbers that have the same ratio as them” instead of “that have the same ratio to each other”.

⁴³ Tarán, *Asclepius* (cit. n. 19), 18; and M. Roueché, “Why the Monad is Not a Number: John Philoponus and *In De Anima* 3”, *JÖB* 52 (2002), 95-133: 116-118, claim that Rec. II was composed in Late Antiquity, but their arguments rest on weak bases.

⁴⁴ On this 14th-century Byzantine scholar see now A. Gioffreda, *Tra i libri di Isacco Argiro*, Transmissions 4, Berlin 2020.

⁴⁵ Tannery, “Rapport” (cit. n. 19), 302; *contra*, Tarán, *Asclepius* (cit. n. 19), 18. The scholia ascribed to Argyros are at Vat. gr. 1411, ff. 49v (nr. 73), 59v (nr. [1]55), 95r (nr. 17), 102v (nr. 53). The former two are edited in F. Acerbi, “I problemi aritmetici attribuiti a Demetrio Cidone e Isacco Argiro”, *Estudios bizantinos* 5 (2017), 131-206, Testo 21; the latter two in Delatte, *Anecdota* (cit. n. 25), 139.21-26 and 173.19-174.3.

⁴⁶ Check I.23.15, Version **D**, Texts **C.2**, **C.5.1E**, **C.5.2**, and see Acerbi, “I problemi” (cit. n. 45).

the side of his own additions. We have also read Argyros' autograph scholium in Text **C.10** above, and it does not coincide with the corresponding exegetical unit of Rec. II.

The last table shows that Rec. IV is not a redaction of Ammonius' lecture notes (or a Byzantine recension of such a redaction) as the other "Recensions" are, but an independently conceived writing. In our case, Rec. IV employs different wordings of crucial statements (red colour above), offers a very long, and different exegesis of the crux in *Ar.* I.23.15 (second cell), and does not even allude to the aporia of the *pythmenes* on which the others spend a rhetorical question.⁴⁷

As our specimen shows, Rec. IV is a commentary-paraphrase that is not organised in "lemmas" as the other Recensions are; we read in it –dismembered among uncountably many repetitions and examples– a fair amount of *Ar.*, so that the argument of Rec. IV can sometimes be followed without having *Ar.* at hand. This is impossible in the other Recensions. Thus, as a literary product, Rec. IV stands mid-way between the other Recensions and Iamblichus' *in Nicomachi Arithmetica*, which is in fact a recension of *Ar.* It is fairly obvious that the author of Rec. IV has largely drawn from the exegetical tradition, the other Recensions included.⁴⁸ There is not the slightest doubt, in my opinion, that Rec. IV is a Byzantine writing, and a fairly typical one because of its prolixity and of its paraphrastic habit; a similar literary product is Nicholas Cabasilas' commentary on Book III of Ptolemy's *Almagest*.⁴⁹

The author of Rec. IV is anonymous. If he has to be a Byzantine scholar, as I think he has to, we might make a case for excluding Michael Psellus –for his name would hardly have disappeared– and the 12th-century scholars who gathered around Anna Comnena, for we read scholia to *Ar.* I.23.15 ascribed to Michael of Ephesus and Eustratius and they are fairly different from the text of Rec. IV (see the next Section).⁵⁰ It is possible that Rec. IV is the product of the renewed interest in philosophy and the sciences that characterized the Nicaean and early Paleologan periods; a *terminus ante quem* is set by the copy of the earliest manuscript witness of the text –Ambros. H 58 sup. (gr. 438; *Diktyon*

⁴⁷ Ammonius summarizes here Iamblichus, in *Nic.* III.52-55, 116.12-29 Vinel.

⁴⁸ Cf. Tarán, *Asclepius* (cit. n. 19), 20.

⁴⁹ On the mathematical skills of this important theologian, see F. Acerbi – I. Pérez Martín, "Les études géométriques et astronomiques à Thessalonique d'après le témoignage des manuscrits: de Jean Pédiasimos à Démétrios Kydônès", *Byzantion* 89 (2019), 1-35: 13-17. Cabasilas' commentary is unedited.

⁵⁰ For a first orientation on 12th-century scholarship, see A. Kaldellis, "Classical Scholarship in Twelfth-Century Byzantium", in Ch. Barber – D. Jenkins (eds.), *Medieval Greek Commentaries on the Nicomachean Ethics*, Leiden – Boston 2009, 1-43. The seminal study on Anna Comnena's circle is R. Browning, "An Unpublished Funeral Oration on Anna Comnena", *PCPhS*, n.s. 8 (1962), 1-12.

42871)– dated to between 1295 and 1315.⁵¹ Only two witnesses of Rec. IV have its text laid out in the form of a framing commentary on *Ar.*, namely, Laur. Plut. 58.29 and Ambros. B 77 sup.; it is obvious that the latter is a conformal copy of the former (check also the apparatus of I.23.15, Versions **A** and **D**).

6.3. The scholia

The exegetical record in form of isolated scholia is exceptionally rich (recall that *Ar.* I.23.15 makes three claims); for this reason, I was induced to create so many **C**-subdivisions. The presence of specific scholia may suggest filiations between manuscripts, but it must be stated clearly that shared scholia may originate either in slavish copy or in collation of another exemplar. Thus, the scholia shared by several witnesses can only provide the basis for a supporting criterion of filiation between manuscripts.

I set out in the following table such scholia as have an author name attached or as are transversal to my **C**-subdivisions; brackets mean either “anonymous” (round), or “penned by a later hand” [square], or “partial copy” {braces}:

ms	Mich.	Dyrr.	Eustr.	Dishyp.	Prot.	Arg.	C.6a	C.14a	C.16a	C.17b	C.18
Ambros. I 8 sup.	(C.9a)										
Ambros. G 62 sup.	C.8a	(C.8b)	C.8c	C.8d							
Athens, EBE 1115	C.9a										
Bucur., BAR gr. 620					C.11a						
Gott. philol. 66					C.11a						
Hamburg, S-UB, philol. 89		{C.13}		{C.13}							
Kharkow, UL 269-p							*		*		
Laur. Conv. Soppr. 30							*		*		*
Laur. Plut. 28.35	(C.7a)				C.11a						
Marc. gr. Z. 309	(C.8a)	(C.8b)					*				
Marc. gr. Z. 316			C.8c							*	
Marc. gr. Z. 318	C.8a	C.8b	C.8c								
Marc. gr. Z. 319		C.8b	C.8c							*	
Marc. gr. Z. 320								*			
Marc. gr. Z. 333	{{(C.8a)}}		(C.8c)		(C.11b)						
Matrit. 4678							*				
Monac. gr. 76					(C.11b)						
Monac. gr. 482	(C.7a)				C.11a						

⁵¹ A. Turyn, *Dated Greek Manuscripts of the thirteenth and fourteenth centuries in the Libraries of Italy*, 2 vol., Urbana – Chicago – London 1972, vol. 1, 88-90.

ms	Mich.	Dyrr.	Eustr.	Dishyp.	Prot.	Arg.	C.6a	C.14a	C.16a	C.17b	C.18
Mutin. α.T.8.14	{[(C.8a)]}		[C.8c]		(C.11b)						
Mutin. α.U.9.7					C.11b						
Neap. III.C.1								*			
Nor. Cent. V. App. 36						C.10					
Oxon. Bodl. Holkh. 71							*		*		*
Par. Coislin 174					C.11b						
Par. gr. 2450								*			
Par. gr. 2480					C.11a						
Par. gr. 2483							*		*		
Par. gr. 2531					C.11b						
Roma Abb. S Paolo 24C					C.11a						
Scorial. P.II.3							*		*		
Scorial. X.I.9			C.8c							*	
Vat. Barb. gr. 273								*			
Vat. gr. 186	(C.9a)										
Vat. gr. 198		{C.13}		{C.13}							
Vat. gr. 256	C.9a										
Vat. gr. 1026							*				
Vindob. phil. gr. 220								*			

As the table shows, particularly successful have been five scholia of assigned author, three of which can be found in a fair number of manuscripts, either anonymous or with ascribed authorship. Two of these five scholia come from the 12th century scholars mentioned above, namely, Michael of Ephesus and Eustratius of Nicaea; two others were composed –apparently in the context of a scholarly debate– by an anonymous reader from the town of Δυρράχιον (present-day Durrës), and by a judge Nicholas Dishypatos, of whom no more information appears to be available (Texts **C.7-9** and **C.13**).⁵² Michael's and Eustratius' exegeses were added by a later hand, partly and misattributed, in Mutin. α.T.8.14, and in its copy Marc. gr. Z. 333 (check also Text **C.11b**); Eustratius' alone also figures in Marc. gr. Z.

⁵² Dishypatos mentions the anonymous scholar from Durrës and dubs him “most Aristotelian”. As far as these two characters are concerned, proximity searches in the TLG give no results that may provide serious grounds for an identification. Tannery, “Rapport” (cit. n. 19), 329, first put these four scholia to scholarly attention; however, he could not read the epithet that identifies the author of the first scholium, which can be found in the Martini-Bassi catalogue. The pieces of information that come from secondary literature have recently been compiled in I. Nesseris, *Η Παιδεία στην Κωνσταντινούπολη κατά τον 12ο αιώνα*, PhD Thesis, University of Ioannina 2014, vol. 1, 286-289. On Michael of Ephesus' mathematical activities see F. Acerbi – B. Vitrac, “Les mathématiques de Michel d'Éphèse”, *REB* 80 (2022).

316 and in its conformal apograph Scorial. X.I.9 (see also Text **C.17b**); the latter two scholia were transcribed in severely abridged form by Malachias in his Vat. gr. 198 (he also calls the judge “Niketas”), and in Hamburg, S-UB, philol. 89 therefrom (check also Texts **C.1** and **C.5.1D**). These four scholia are compiled together only in Ambros. G 62 sup., a very important witness of *Ar.*, which alone hands Dishypatos’ text down to us in its longer version; the four Bessarion manuscripts Marc. gr. Z. 309, 316, 318 (a “patrimonial” manuscript), 319 offer a different text of those of the above scholia they happen to preserve.

As for the fifth scholium in this group, it was in fact the most successful scholium on one single claim of *Ar.* I.23.15, and was eventually included in some Rec. I manuscripts; in most of them, it is ascribed to some Theodoros/Demetrios Protocensor (Text **C.11**). Theodoros’ and Michael of Ephesus’ scholia, the latter here anonymous, were transmitted from Monac. gr. 482 to its copy Laur. Plut. 28.25 (check also Texts **C.1**, **C.5.1A**, and **C.7**). In the former manuscript, Theodoros’ scholium is attached to Rec. I, which was penned by Neophytos Prodromenos as a whole after *Ar.*; Michael’s annotation was instead located in due place at the end of Book I of *Ar.*; the copyist of Laur. Plut. 28.35 re-located Rec. I in the margins of *Ar.*, so that he eliminated Michael’s scholium while keeping Theodoros’. The other manuscripts that assign the scholium to Theodoros all stem from Roma, Arch. S. Paolo 24C (cf. Section **6.1** above). The ascription to Demetrios was spread from Mutin. α.U.9.7 to its (quite incorrect) conformal copy Par. gr. 2531 (have a look at the layout at the end of Book I) and, most notably, to some Rec. I manuscripts, among which I have checked Angel. gr. 1 and BNCF, Fondo Naz. (*olim* Magliab.) II.III.37 and Vat. gr. 2262. The scholium lies anonymous in Mutin. α.T.8.14 and in its copy Marc. gr. Z. 333 (see also Text **C.8ac**); the text is of the Demetrios-type.

Nikephoros Gregoras apposed glosses, a scholium (Text **C.14a**) and diagrammatic scholia to *Ar.* I.23.15 in his own recension manuscript Marc. gr. Z. 320. We find these items in the direct copies Ambros. P 121 sup. (only the diagrams), Par. gr. 2450 (a conformal apograph of which is Vat. Barb. gr. 273), Vindob. phil. gr. 220 (a copy of which is Neap. III.C.1); see also the transmitted correction in Texts **C.5.1AF**. Isaak Argyros’ scholium (Text **C.10**) is instead unique to his own fair copy Norimb. Cent. V. App. 36. A distinctive reading (Adjunct, Version F) and diagrams show that Argyros used Par. gr. 2107 as his model; a conformal apograph of the Paris manuscript is also Guelf. 36 Gud. gr. (cf. **C.5.1** gloss).

As for anonymous scholia, Text **C.6a** is shared by Matr. 4678 and Vat. gr. 1026; together with Text **C.16a**, it is also shared by Kharkow, UL 269-p, 369-c and by its copies Par. gr. 2483 and Scorial. P.II.3. These two texts together with Text **C.18** are further shared by the magnificent Oxon. Bodl. Holkham 71 and by its likely copy Laur. Conv. Soppr. 30. The Kharkow manuscript shares with Oxon. New Coll. 299 and with Vat. gr. 1051 and its conformal copy Par. gr. 2481 a distinctive mistake in *Ar.* I.23.15 (Version E).

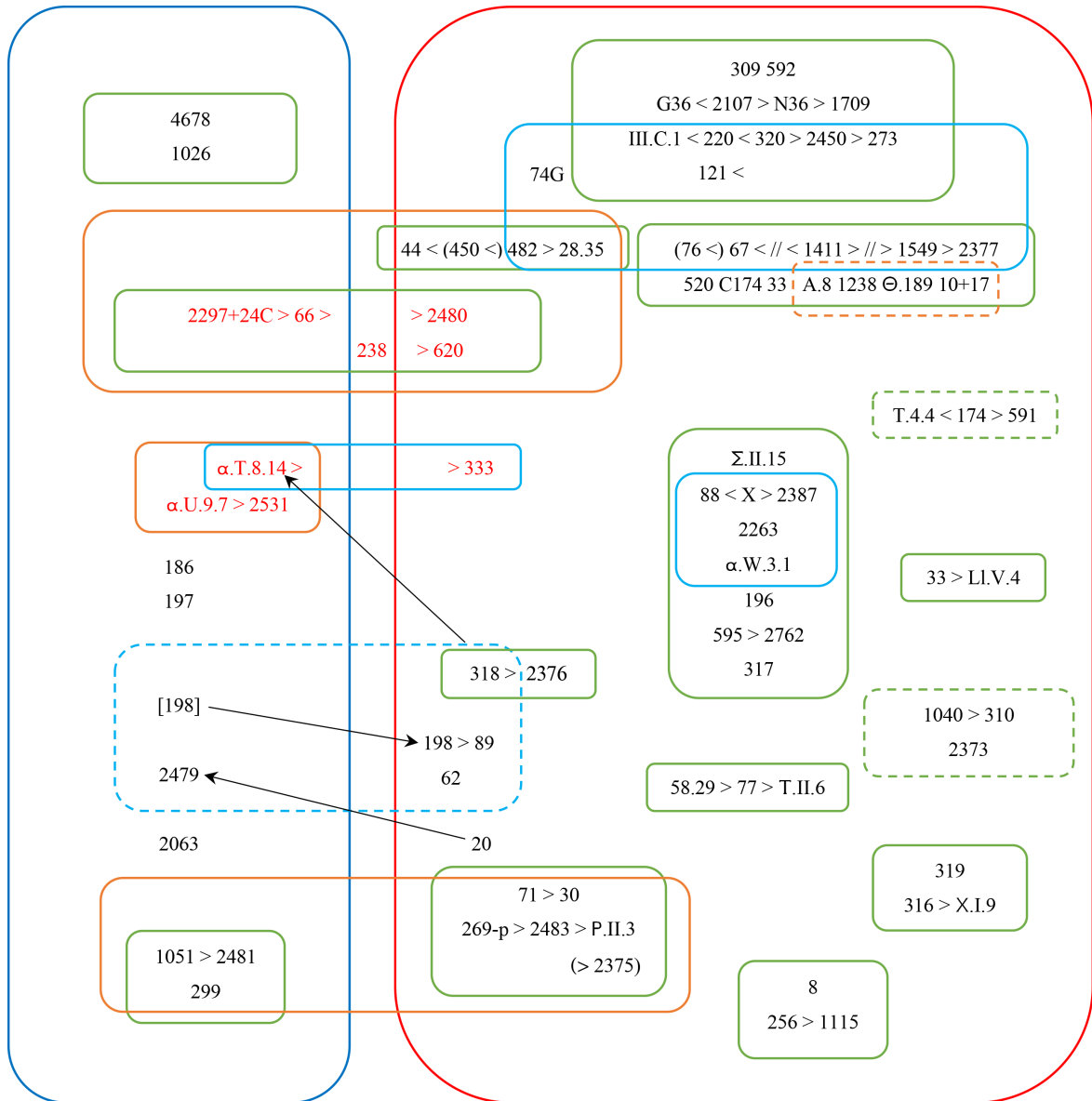
The following table sets out the scholia to the Adjunct:

ms	Text			
Ambros. I 8 sup.		C.9bd		
Athens, EBE 1115		C.9bd		
Berol. Phillipps 1549	C.5.2			
Cizensis 67	C.5.2			
Hamburg, S-UB, philol. 88			C.12	
Marc. gr. Z. 317			C.12	
Marc. gr. Z. 595			C.12	
Monac. gr. 76	C.5.2			
Oxon. Bodl. Selden Supra 20		C.9bd		
Oxon. Lincoln Coll. gr. 33	C.5.2			
Par. gr. 2373				[C.15]
Par. gr. 2377	C.5.2			
Par. gr. 2479		[C.9bd]		
Par. gr. 2762			C.12	
Vat. gr. 256		C.9bd		
Vat. gr. 1411	C.5.2			
Vat. gr. 2387			C.12	

The second column singles out all representatives of Rec. II I was able to check. As said, they all stem from Vat. gr. 1411. As for the third column, the indicated scholia and related material (inclusive of Michael's scholium **C.9a**) were transmitted by conformal copy from Vat. gr. 256 to Athens, EBE 1115 (check also Text **5.C.1B**), and by collation, along with the Adjunct itself **C.5.1B**, from Oxon. Bodl. Selden Supra 20 to Par. gr. 2479 (here a later hand). The fourth column displays a group of tightly related manuscripts, further linked by the mistake in I.23.15, Version C.

The information gathered in this Section and an analytic look at the *varia lectio* recorded in Section 5 (this is summarized in the tables of the Appendix) allow outlining the following schematization of a part of the textual tradition of *Ar*. The two large continents, connected by several bridges, represent the presence (absence on the left) of the Adjunct; the sign ">" connects an apograph to its (likely or obvious) model. The manuscripts are identified by the number that characterizes their signature; those marked in red have Theodoros/Demetrios Protocensor's scholium. Enclaves delimited by broken lines also depend on extratextual criteria.⁵³

⁵³ These are: presence of the "Soterichos" scholium located between the two Books of *Ar*. (blue); Rec. II ascribed to Proclus (orange); diagrammatic apparatus (green: see Acerbi,



7. Exegetical stances

It is important to recall that *Ar. I.23.15* makes three claims, whose meaning has been explained in Section 3 above. How these claims, along with the context-setting clause, were interpreted by the scholiasts is best discussed by taking the Adjunct as a reference; in the following table, the claims of the Adjunct are singled out, and *I.23.15* is set out facing and disassembled (cf. Section 6.1 above, where the complementary operation is performed):

“Eliminazioni” [cit. n. 3]). See the Appendix for details. On the “Soterichos” scholium see C. Hofstetter, “Le mystérieux Soterichos: contribution à l’établissement de l’histoire du texte d’un commentaire byzantin à la psychogonie de Platon”, *RHT* 13 (2018), 59-91.

Adjunct	Ar. I.23.15	
ἐπὶ πασῶν μέντοι τῶν ἐκκειμένων ἐκθέσεων	ἐπὶ πασῶν δὲ τῶν διαζευχθεῖσων	1
πάντως οἱ ἄκροι τετράγωνοι εἰσιν, οἱ δὲ μέσοι ἐκ τῶν πλευρῶν αὐτῶν ἐπ' ἀλλήλαις γενομένων,	πάντως δὲ οἱ ἄκροι τετράγωνοι	5
καὶ ὁ μὲν πρῶτος τῆς ἀπογεννώσης εἰς τὸν ἐλάττονα τῆς γινομένης μεταβαίνει,	ὁ δὲ πρῶτος εἰς τὸν ἐλάττονα μεταβαίνει,	4
ἐν ἀμφοτέραις δὲ ταῖς γεννηθείσαις	καὶ ἀφ' ἧς ἀμφοτέραις,	2
ὁ ἔσχατος καὶ μείζων τετράγωνος ὁ αὐτός ἐστιν	ὁ μὲν ἔσχατος τετράγωνος ὁ αὐτὸς μένει,	3

Thus, as seen in Section 3 above, two “disjoined” numerical means are “generated” from an assigned mean (this is the Context: rows nr. 1-2 above): the extreme terms of these means are “always square numbers” (Claim 1: row nr. 5 above), the “first” (and largest) term of the “generating” mean “is transformed into the smaller <term> of the resulting <mean>” (Claim 2: row nr. 4 above), the “last and largest” term is the same in the resulting means (Claim 3: row nr. 3 above). In the Adjunct, we also get for free an enrichment of Claim 1: the middle term of a geometric mean is the geometric mean of the extremes. What is at issue here, then, are the numerical means and their terms. No mention of ratios, which, however, are the backbone of the general context of *Ar. I.23.6-17* in virtue of the identification of ratios and means,⁵⁴ as seen in Section 2 above. This imperfect match could not be swallowed by several readers, who insisted in forcing *Ar. I.23.15* in the unsuitable mould of ratios. If an unsuitable mould might well prove harmless in point of exegesis, the real issue in interpreting *I.23.15* is simple: which of the two extremes is “the first”, and which is “the last”. The alternative is not settled by the wording of *I.23.15*, as we have repeatedly seen. Moreover, it is clear that all of the following constraints cannot be filled at the same time: (1) the “first” and the “last” term cannot have the same position in a mean; (2) all Claims must refer to both “resulting” means; (3) if something within a specific kind is “transformed into <anything> smaller” in the same kind, it cannot be the smallest such item around. Add to all of this the diabolic touch of interchanging the extremes of the mean in the shift from the “direct” to the “inverse” arm of the prescription. The result is the exegetical diffraction I shall summarize in what follows.

Let us start with the Context. Most interpreters agree in the above interpretation, usually by conceiving of what is disjoined as means-as-ratios (σχέσεις) or means-as-proportions (ἀναλογίαι): see Texts **C.1**, **C.2**, **C.4** (Rec. I, II, IV), **C.5.1D** (a variant reading in the Adjunct), **C.7** gloss, **C.8c** and gloss, **C.8bd** = **C.13ab**, **C.9b**, **C.10** and gloss, **C.12**

⁵⁴ When defining the means at *Ar. I.23.7*, Nicomachus calls them ἐκθέσεις “setting-outs” (wiz., of numerical terms), and this denomination will be picked up in the Adjunct.

gloss, **C.14a** and gloss, **C.16b**, **C.18** gloss. The exceptions are Texts **C.3** (Rec. III), which does not tackle this point, Texts **C.6** glosses, **C.7a** = **C.8a** = **C.9a**, which quite incorrectly think that what is disjoined are terms such as the unit, the dyad, etc., and **C.16** gloss, which identifies “disjoined” with “inverse”. Finally, note that the Adjunct disconnects the two phrases that define the Context, and distributes them between the Claims.

As for the Claims, let us start on safe ground, namely, with Claim 1. Rec. I-IV (Texts **C.1-4**) shift from a cautious “proved by means of lines (= rigorously)” to a questionable “proved by the geometer/in geometry”, cite a pseudo-Euclidean enunciation (modified by Rec. II, as seen in Section 6.2; the formulation of Rec. I/III is also adopted in Texts **C.7b** and **C.9c**, **C.17a**, **C.18**), and offer one or more basic examples. Rec. I-III, along with Texts **C.6b** (very sketchily), **C.17a**, **C.18**, and, in a different formulation, Argyros’ scholium **C.10**, also present and solve the aporia of the *pythmenes*: the property holds for means in lowest terms, but each mean contains two instances of the ratio it corresponds to, and neither instance can be in lowest terms (cf. Section 2 above), that is, those terms that are the *pythmenes* of the ratio.⁵⁵ The scholium to the Adjunct **C.5.2** explains, by quoting the enunciation of *Elem.* VIII.11, that the middle term referred to by the Adjunct is the product of the “sides” (= square roots) of the extremes, whereas Texts **C.9d**, **C.12b**, **C.14** gloss to the Adjunct, **C.15**, **C.16ab**, **C.17c** present a series of examples of this very last statement. Texts **C.8**, **C.11**, **C.13** (Malachias’ rewriting of **C.8db**), **C.14**, **C.15** do not discuss this Claim.

Interpreting Claim 2 was a thorny issue. Rec. III (Text **C.3**) is silent on it. Philoponus (Text **C.1**), who could not plagiarize anyone, concocts an explanation that is just a smoke screen: “the first drifts towards the unit, for it also originated from the unit”. The reviser of Rec. II (Text **C.2**) cannot be happy with this and offers a correct explanation; he adopts, however, a terminology that others also applied in this context: the largest term of a mean is read as the “antecedent” of the ratio associated to it, the smallest term of the mean being associated with the “consequent” of the “same” ratio. As explained in Section 2, the two terms are “antecedent” and “consequent” of *different* albeit equivalent ratios, neither of which is in lowest terms. This fact slightly complicates matters. The smaller-term reading can be found in Texts **C.8d** = **C.13a**, **C.9b**, **C.10**, **C.12a**, **C.15** gloss, **C.16b**, **C.17** gloss; the terminology of “antecedent” and “consequent”, which obviously also fits Claim 3, is further adopted in Texts **C.8c**, **C.8d** = **C.13a**, **C.10**, **C.11**, **C.17** glosses.

Rec. IV (Text **C.4**) introduces a major misconception to us: the first term of the generating mean is not transformed into a smaller term but into a smaller *ratio*. As said at the

⁵⁵ In general, a *pythmen* is the smallest number, or set of numbers, or ratio, to which a certain property applies. The concept was already well-developed in the times of Plato: *R.* 546c1-3.

end of Section 2, this can only be true, and not even in every instance, for the “inverse” arm of the prescription, contrary to the explicit statement of *Ar.* I.23.15 that the property must hold for both arms. We also find the ratio-reading in Texts **C.6a** and glosses, **C.7a** and glosses = **C.8a** = **C.9a**, **C.8c**, which however combines it with the smaller-term reading, **C.8** gloss, **C.12** glosses, **C.14a** and glosses, **C.16a**, **C.18** gloss.

Text **C.8b** appears to endorse the smaller-term interpretation but gets entangled in an unwelcome confusion because it writes the generating mean of its own basic example in reverse order: 9,6,4 instead of 4,6,9. Of course, this move has the effect of inverting the referent of the pronouns “first” and “last” in *Ar.* I.23.15. This problem disappears in Malachias’ radical rewriting in Text **C.13b**, where the numbers of the example are also different.

Let us finally come to Claim 3. The situation with Rec. III (Text **C.3**) and Rec. I (Text **C.1**) is the same as above: the former is silent, the latter deploys a smoke screen. Rec. II (Text **C.2**) adopts two basic interpretations: (1) Claim 3 applies to both arms of the prescription and not to the direct arm only (see the discussion in Section 2 above); (2) as a consequence, the “last” term is the smallest one in a mean, or, by using the wording of Rec. II, is the “consequent” of the lesser of the two ratios that figure in the mean. In its not-so-clear-cut formulation, Rec. IV seems to put forward both (1) and (2). The same position as Rec. II we find in Texts **C.7** glosses, possibly (the line of argument is very confused; one must induce from the interpretive stance for Claim 3, but note that the final remark of Text **C.8d** seems to imply the greatest-term reading) in **C.8b**, and in **C.8cd**, **C.10**, **C.11**, **C.12a**, **C.16a**.

The Adjunct **C.5.1** adopts (1) above but also the reading that (3) the “last” term is the greatest one in a mean: as a consequence, the two greatest terms in the resulting means are identical (this is also the reading I prefer). The same position we find in Texts **C.6b** glosses, **C.7a** = **C.8a** = **C.9a**, in Malachias’ radical rewriting of **C.8b** in Text **C.13b**, and again in Texts **C.9b**, **C.16b**, **C.17** glosses. Texts **C.8d**, and **C.14** and glosses appear to set forth the idea that the last square remains the same as a square (“as for quality”), not as of its numerical value (“as for quantity”).

Texts **C.6a**, **C.13a** (Malachias’ rewriting of **C.8d**), **C.15** do not really tackle the issue.

All in all, we may be charitable with our commentators and scholiasts: *Ar.* I.23.15 *really is* an ambiguous sentence. The scholia that definitely beat around the bush are a handful, among which Philoponus’ Rec. I. Noteworthy points are: Malachias’ rewritings of Dishypatos’ and the man from Dyrrachios’ scholia; the clear hints of a scholarly debate lingering over *Ar.* I.23.15; the metonymic misinterpretation that makes, for instance, Michel of Ephesus think that the referents’ set of the pronouns “last”, “first”, and

“smallest” may contain both terms and ratios; the systematic identification of means and ratios, refined in some scholiasts by the use of the technical terms “antecedent” and “consequent”; the moderate use of numerical examples; the citation of a phantom-theorem in the *Elements*.

8. Conclusion

Let us finally sketch, by referring to the three aims listed at the beginning of this paper, the main results of this *tour de force*.

Philological aim 1. Some order has been made in the textual tradition of *Ar*. It is shown that the tradition is rooted in an archetype to be located between Iamblichus and Ammonius because of the presence, in all manuscripts, of the interpolation *Ar*. I.23.15. It is also shown that a text I have called “the Adjunct” and that is witnessed to by most manuscripts is a clarification of *Ar*. I.23.15 that has crept into the text, certainly after Ammonius. Some of these manuscripts allow following the process of absorption of the Adjunct by the text. The *varia lectio* associated to I.23.15 and to the Adjunct, along with the pattern of presence and the *varia lectio* of the scholia thereon, allow outlining a first map of the witnesses of *Ar*. Yet, much of the *varia lectio* perfectly fits the syntactical and mathematical context: this shows how difficult is to apply the genealogical method in the topmost portion of the stemma of *Ar*.

Philological aim 2. The relationships between the so-called Recensions of Ammonius’ lecture notes are settled once and for all: only Rec. III and Rec. I deserve this appellation; Rec. II is a Byzantine recension of Rec. I; Rec. IV is an independently conceived writing of Byzantine origin. As for Rec. III and Rec. I, my impression is that Philoponus simply reworked Asclepius’ set of scholia, with a close look at his own Nicomachus. It is of some interest, and even slightly bewildering, that Ammonius/Asclepius did not comment on the most problematic claims of *Ar*. I.23.15.

Historical aim. Byzantine scholars hotly debated about the meaning of *Ar*. I.23.15 and of the Adjunct; the exegetical documentary record is very rich, some scholia being transmitted in fairly different redactions. The scholarly debate maybe found its *akmē* in the 12th century, maybe within the circle of scholars gathered around Anna Comnena (after all, they had to study some number theory for commenting on Aristotle’s *Nicomachean Ethics*), and could be taken to be responsible for some of the pre-traditional modifications that affect Nicomachus’ text and for part of the rich apparatus of scholia in form of syllogistic diagrams that characterizes some manuscript witnesses.⁵⁶

⁵⁶ See Acerbi, “Eliminazioni” (cit. n. 3), on this.

Appendix. The documentary record

MANUSCRIPTS OF NICOMACHUS, *INTRODUCTIO ARITHMETICA*, THAT CONTAIN THE END OF BOOK I (83)

The four central columns set out the Version of *Ar.* I.23.15, the number of the Text number of the associated scholium, if any, the Version of the Adjunct, the presence or absence of glosses to I.23.15, respectively. The last two columns give the Hoche's and D'Ooge's *sigla*. Later hand texts are within brackets; within braces are texts that can only be read partly.

ms	copyist	date	Book I	Book II	I.23.15	Txt	Adj	gl	remarks	H	D
Ambros. I 8 sup. (gr. 451) (<i>Diktyon</i> 42887)		12th ex. – 13th in.	1v-34r	34r-73r	B	9	B	Y	inc 4.7 αὔτη marg sch sch [Michael of Ephesus] related to Vat. gr. 256		
Ambros. B 77 sup. (gr. 105) (<i>Diktyon</i> 42327)		14th ex. – 15th in.	2r-58r	58v-101r	D	3 4	A	N	1r-58r, 58v-101v Rec IV 102r-132r, 132r-149v des mut Rec III copy of Laur. Plut. 58.29 as of <i>Ar</i> + Rec IV		
Ambros. G 62 sup. (gr. 404) (<i>Diktyon</i> 42822)		13th ex. – 14th in.	28r-43v	48r-65v	A	8	A	N	sch Michael of Ephesus, [Δυρραχίου], Eustratius, Dishypatos Soterichos splits <i>Ar</i> .		
Ambros. P 121 sup. (gr. 640) (<i>Diktyon</i> 43117)		15th	111r-130v	130v-152v	A	N	F	N	copy of Marc. gr. Z. 320		
Athens, EBE 1115 (<i>Diktyon</i> 3411)		14th ex.	1-59	60-134	B	9	{B}	N	marg mix comm sch Michael of Ephesus copy of Vat. gr. 256 partial lacuna on p. 59 affects Adj		A
Bonon. BU 2263 (<i>Diktyon</i> 74179)		16th	1r-16v	17r-34r	C	N	B	N	many diagrams m rec related to Hamburg, S-UB, philol. 88 and Vat. gr. 2387		

ms	copyist	date	Book I	Book II	I.23.15	Txt	Adj	gl	remarks	H	D
Berol. Phillipps 1549 (<i>Diktyon</i> 9450)	Valeriano Albini	16th ²	1r-84v	89r-159r	D	2 5.2	E	N	1r-88v, 89v-159v Rec II incl <i>Ar</i> sch ἐν ἄλλοις stems from Vat. gr. 1411 after rec		
Bucuresti, BAR gr. 520 (Litzica 210) (<i>Diktyon</i> 10593)		18th	87-118	119-154	D	N	E	N	nearly all diagrams missing		
Bucuresti, BAR gr. 620 (Litzica 183) (<i>Diktyon</i> 10693)		18th	11-234	235-430	A	1 11a	A	N	sch Theodoros protocensor conformal copy of Gott. philol. 66		
Erlangen, UL A.8 (<i>Diktyon</i> 14298)		17th	1r-39r	39r-54v	D	N	E	N	s.l. Rec II anchoring numbers Rec II (missing) ascr. Proclus stems from Vat. gr. 1411		38
Gott. philol. 66 (<i>Diktyon</i> 17413)		13th ex.	1r-140r	142r-265r	A	1 11a	[A] [A]	N	1r-141r, 142r-266r Rec I sch Theodoros protocensor Adj in Rec I, Adj m rec copy of Roma, Arch. S. Paolo 24C	G	G
Guelf. 36 Gud. gr. (<i>Diktyon</i> 72080)		16th in.	1r-17v	17v-32v	A	N	F	N	Guelf. 43 + 37 + 36 + 40 Gud. gr. annotated by Matteo Macigni systematic copy of Par. gr. 2107	Γ	Γ
Hamburg, S-UB, philol. 88 (<i>Diktyon</i> 32406)	Alexios	ca 1294	1r-24r	24v-52v	C	12	B	Y	sch; cf. Vat. gr. 2387 no compl of Vat. Barb. gr. 164		37
Hamburg, S-UB, philol. 89 (<i>Diktyon</i> 32407)	Francesco Gozzadino	1633	1-76	77-152	A	1 13	[D]	N	161-373, 399-548 Rec I sch Dishypatos; sch Διππαχίου Adj marg Soterichos splits Rec I conformal copy of Vat. gr. 198	H	H

ms	copyist	date	Book I	Book II	I.23.15	Txt	Adj	gl	remarks	H	D
Kharkow, UL 269-p, 369-c (<i>Diktyon</i> 12442)		14th a.m.	67r-87v	88r-102v	E	6a 16	B	Y	sch; sch ἐν ἄλλοις 102v <i>Def.</i> 138.9-10+ lac 48.4 ἀρι]θμῶ – 53.23 προχωροῦσιν, 63.1 πολλαπλασίον – 67.19 καὶ, 75.16 ψυχογονίαν – 81.11 ζ, 107.7 ὀκτάκις – 119.21 τε, 123.15 καὶ – 126.10 ἴσον after ff. 83, 86, 88, 95, 96, resp correct folios order 97, 99, 98, 101, 100		
Laur. Conv. Soppr. 30 (<i>Diktyon</i> 15795)		13th ex.	9v-43r	43v-68r	A	6a 16a 18	A	Y	sch; several hands annot Nikephoros Gregoras prol m rec des. 77.5 περὶ possibly a copy of Oxon. Bodl. Holkham 71		
Laur. Plut. 28.35 (<i>Diktyon</i> 16216)		14th	2r-31v	31v-44v	A	1 11a	A	Y	des. 116.9 διατηρήσουσι 2r-30r, 30r-50r Rec I sch Theodoros protocensor copy of Monac. gr. 482		
Laur. Plut. 58.29 (<i>Diktyon</i> 16447)		14th ²	97r-153r	154r-196v	A	4	A	N	95v-196v Rec III I.α + Rec IV Psellos [cf. Soterichos] indep. of <i>Ar.</i>		
Leid., BRU, Periz. Q° 39 (<i>Diktyon</i> 37878)		14th	1r-23v	23v-40v	B	N	B	N	incomplete set of diagrams several hands		39
Leid., BRU, BPG 74G (<i>Diktyon</i> 37728)		16th a.m.	4r-25r	25v-48r	D	N	F	N	sch; 53v-65r Rec I extr related to Par. gr. 2450		
Leiden, BRU, Voss. gr. Q° 23 (<i>Diktyon</i> 38130)		14th a.m. 15th	1r-30r	31r-62v	A	N	N	N	lac 140.9 καὶ – 143.11 οἶον des 146.17 διὰ τεσσάρων		
Marc. gr. Z. 309 (coll. 300) (<i>Diktyon</i> 69780)	Philotheos of Selymbria	14th m.	183v-193v	193v-204v	A	6 8ab	F	Y	sch [Michael of Ephesus], [Δυρραχίου] possibly related to Par. gr. 2107		

ms	copyist	date	Book I	Book II	I.23.15	Txt	Adj	gl	remarks	H	D
Marc. gr. Z. 316 (coll. 670) (<i>Diktyon</i> 69787)		14th a.m.	5r-61v	62r-126v	A	8c 17	A	Y	prol des. 77.5 περὶ sch Eustratius 129r-244v, 245r-340r Rec I		
Marc. gr. Z. 317 (coll. 637) (<i>Diktyon</i> 69788)		14th m.	1r-23v	23v-49v	B	12	A	N	no comm related to Marc. gr. Z. 595		
Marc. gr. Z. 318 (coll. 994) (<i>Diktyon</i> 69789)		14th ²	1r-21r	21r-44v	A	8abc	A	Y	sch Michael of Ephesus, Δυρραχίου, Eustratius Soterichos indep. of Ar. possibly related to Ambros. G 62 sup. and Vat. gr. 198		
Marc. gr. Z. 319 (coll. 1029) (<i>Diktyon</i> 69790)		13th m.	1r-39r	40r-75v	BD	8bc 17b	A	N	sch Δυρραχίου, Eustratius		
Marc. gr. Z. 320 (coll. 638) (<i>Diktyon</i> 69791)		1320-30	19r-52v	53r-92v	A	14	A	Y	Gregora's recension manuscript		
Marc. gr. Z. 330 (coll. 1029) (<i>Diktyon</i> 69801)		14th a.m.	288r-302r	302r-314v	B	N	A	N	no sch		
Marc. gr. Z. 333 (coll. 644) (<i>Diktyon</i> 69804)	Bessarion	ca 1426	39v-61v	62r-82v	A	1 4part 8ac 11b	A	N	39r-61v, 62r-83r mix sch sch [Demetrios] protocensor [Michael of Ephesus], [Eustratius] final sentence Rec IV copy of Mutin. α.T.8.14 as of sch		
Marc. gr. Z. 514 (coll. 771) (<i>Diktyon</i> 69985)		14th ex.	6r-24v	25r-44r	A	N	A	N	no comm, no sch, no diagr		
Marc. gr. Z. 592 (coll. 529) (<i>Diktyon</i> 77063)	Angelos Vergetios	16th m.	1r-37r	37v-75v	A	N	F	N	no comm, no sch, no diagr related to Par. gr. 2107		
Marc. gr. Z. 595 (coll. 806) (<i>Diktyon</i> 70066)		14th ex.	7r-32v	32v-61r	B	12	A	Y	no comm related to Marc. gr. Z. 317		
Matrit. 4678 (<i>Diktyon</i> 40155)		1060-80	4r-26v	27r-57v	A	6	N	N	sch 4-8 13th p.m. ad 25.9 πολλα- 4r sch Chortasmenos long dittography at end Book I		

ms	copyist	date	Book I	Book II	I.23.15	Txt	Adj	gl	remarks	H	D
Monac. gr. 76 (<i>Diktyon</i> 44520)	Bartolomeo Zanetti 1-93 Camillo Zanetti 277-453 main 5 Mon.27 94-276	ca 1550	220r-248r	248r-276r	D	N [1] [4] [5.2] [11b]	E	N	1r-61r, 62r-93v Rec III I.α + Rec IV 94r-160v, 161r-220r Rec I+ sch [Demetrios] protocensor likely partial copy of Cizensis 67 likely partial copy of Marc. gr. Z. 397	μ	μ
Monac. gr. 482 (<i>Diktyon</i> 44930)	Rec. I Neophytos Prodromenos	13th p.m. 14th m.	96r-133r	133v-169v	A	[1] 7 [11a]	A	Y	173r-220r, 220v-252v Rec I sch [Michael of Ephesus] sch Theodoros protocensor in Rec I	S	S
Mutin. α.T.8.14 (Punt. 105) (<i>Diktyon</i> 43425)	marg Ioannes Chortasmenos	14th 1397-1402	1r-28r	28r-54v	A	1 [8ac] 11b	N	N	1r-28r, 28v-54v Rec I incl <i>Ar</i> sch [Demetrios] protocensor m rec sch [Michael of Ephesus], {Eustratius} integr m rec Rec I almost no primary diagrams		
Mutin. α.U.9.7 (Puntoni 56) (<i>Diktyon</i> 43474)		14th 15th in.	1v-38v	39r-75r	A	1 11b	[A]	N	marg Adj Rec I 1v-39r, 39r-75r Rec I sch Demetrios protocensor ff. 1-12 15th in. (ad I.7.5), diff. model correct folios order 11, 8-10 lac 8.8 καθὰ – 9.9 ἀριθμητικῇ		
Mutin. α.W.3.1 (Punt. 245) (<i>Diktyon</i> 43532)	Paulos	15th	1r-18r	18r-37v	C	N	B	N	38r-88v, 89r-107v Rec IV no diagrams related to Hamburg, S-UB, philol. 88 and Vat. gr. 2387		
Neapol. III.C.1 (<i>Diktyon</i> 46277)	Petrus Creticus	ca 1433	1r-20v	21r-41v	A	14	F	N	copy of Vindob. phil. gr. 220		
Neapol. III.C.6 (<i>Diktyon</i> 46282)		14th m.	1r-32r	32r-61r	B	N	A	N	some diagrams possibly thorn away		

ms	copyist	date	Book I	Book II	I.23.15	Txt	Adj	gl	remarks	H	D
Norimb. Cent. V. App. 36 (<i>Diktyon</i> 46676)	Isaak Argyros	ca 1370	1r-18v	18v-37v	A	10	F	Y	sch copy of Par. gr. 2107	N	N
Oxon. Bodl. Holkham 71 (<i>Diktyon</i> 48139)		13th	265r-278v	279r-293v	A	6a 16a 18	A	N	sch related to Laur. Conv. Soppr. 30		
Oxon. Bodl. Laud. gr. 44 (<i>Diktyon</i> 48266)		16th ex.	25-90	91-188	A	N	A	N	inc mut 17.18 ἐκθέσει προχειρισμένων almost no sch or diagrams in Book I copy of Par. suppl. gr. 450		
Oxon. Bodl. Selden Supra 20 (<i>Diktyon</i> 48465)		14th	40r-43v, 1r-17r	17r-39r	B	9bd	B	N	8.10 εὐ]χρηστά – 18.5 σωρηδόν, 22.19 δὲ usque ad finem note correct folios order		42
Oxon. Lincoln Coll. gr. 33 (<i>Diktyon</i> 48687)		16th	68r-110r	111r-143v	D	2 5.2	E	N	68r-110v, 111r-143v Rec II incl <i>Ar</i> stems from Vat. gr. 1411		L
Oxon. New Coll. 299 (<i>Diktyon</i> 48768)		15th	52r-55v	55v-59r	E	N	N	N			O
Par. Coislin 174 (<i>Diktyon</i> 49313)		14th, 15th	41r-58v	59r-79r	A	N [1] [11b]	F [A]	N	81r-103r, 103r-118v Rec I (15th) Adj <i>Ar</i> et marg Rec I sch Demetrios protocensor		P ₉
Par. gr. 2063 (<i>Diktyon</i> 51692)		14th	9v-33v	34r-55r	D	N	N	N	des. 140.17 αὐταῖς		
Par. gr. 2107 (<i>Diktyon</i> 51736)		14th	59r-85r	85r-112v	A	N	F	N			
Par. gr. 2372 (<i>Diktyon</i> 52004)	John Chionopoulos	15th a.m.	1r-26r	26r-54r	B	N	B	N	almost no diagrams prol follows <i>Ar</i>		P ₁₁
Par. gr. 2373 (<i>Diktyon</i> 52005)		14th	1r-17r	17r-35r	B	[15]	A	Y	sch m rec related to Vat. gr. 1040		P ₈
Par. gr. 2374 (<i>Diktyon</i> 52006)	John Hydruntinos	16th	1r-21v	22r-42v	A	N	D	N	very peculiar diagrams; no sch possibly related to Cantab. UL Kk.V.28		P ₇

ms	copyist	date	Book I	Book II	I.23.15	Txt	Adj	gl	remarks	H	D
Par. gr. 2376 (<i>Diktyon</i> 52008)		16th a.m.	57r-78v	78v-101v	A	3	A	N	1r-34r, 34r-56r Rec III, for which conformal copy of Monac. gr. 431 <i>Ar</i> conformal copy of Marc. gr. Z. 318 many syll diagrams		P ₅
Par. gr. 2377 (<i>Diktyon</i> 52009)		16th	1r-84v	89r-160r	D	2 5.2	E	N	1r-88v, 89v-161v Rec II incl <i>Ar</i> sch ἐν ἄλλοις conformal copy of Berol. Phill. 1549		36
Par. gr. 2450 (<i>Diktyon</i> 52082)	anonymus G	ca 1335	92r-111v	111v-131v	A	14	F	Y	1 sch m rec Gregoras' recension		
Par. gr. 2479 (<i>Diktyon</i> 52111)		12th p.m.	1r-92v	110r-201v	A	[9bd]	[B]	N	m rec marg sch & marg Adj Soterichos splits <i>Ar</i> collated to Oxon. Bodl. Selden Supra 20		P ₄
Par. gr. 2480 (<i>Diktyon</i> 52112)	Ioannes from Siatista	ca 1733	1-345	1-319	A	1 11a	A [A]	N	1-346, 1-319 Rec I sch Theodoros protocensor Adj also in Rec I copy of Gott. philol. 66		P ₃
Par. gr. 2481 (<i>Diktyon</i> 52113)		15th	1r-19r	20r-24v, 41r-53v	E	N	N	N	conformal copy of Vat. gr. 1051		P ₂
Par. gr. 2483 (<i>Diktyon</i> 52115)		14th	5-153	154-313	E	1 6a 16	B	Y	3-154, 154-293 Rec I+ sch ἐν ἄλλοις 315-317 <i>Def.</i> 138.9-10+ copy of Kharkow, UL, 269-p, 369-c as for <i>Ar</i>		P ₁
Par. gr. 2531 (<i>Diktyon</i> 52163)	Michael Souliardos	15th ex.	42r-80v	81r-124r	A	1 11b	[A]	N	marg Adj marg Rec I (part) sch Demetrios protocensor copy of Mutin. α. U.9.7		
Par. gr. 2762 (<i>Diktyon</i> 52398)		15th a.m.	13r-41v	41v-73v	B	12	A	N	related to Vat. gr. 196 possibly a copy of Marc. gr. Z. 595		

ms	copyist	date	Book I	Book II	I.23.15	Txt	Adj	gl	remarks	H	D
Par. suppl. gr. 450 (<i>Diktyon</i> 53192)		15th	6r-35r	35v-68v	A	N	A	N	almost no sch or diagrams in Book I (indirect and partial) copy of Monac. gr. 482		P ₁₀
Scorial. P.II.3 (gr. 23) (<i>Diktyon</i> 15295)		16th p.m.	262r-437r	437r-594v	E	1 6a 16	B	N	262r-437r, 437r-594v Rec I+ incl. <i>Ar.</i> copy of Par. gr. 2483		
Scorial. Σ.II.15 (gr. 95) (<i>Diktyon</i> 15366)		13th ex.	160r-171v	/	A	N	A	N	several hands		e ₄
Scorial. Υ.III.12 (gr. 282) (<i>Diktyon</i> 15495)		14th	1r-21r	21r-42r	A	N	D	N	Adj singled out as such many diagrams missing in indent		e ₅
Scorial. X.I.9 (gr. 351) (<i>Diktyon</i> 14967)		1544	4r-48r	49r-91v	A	8c 17	A	Y	prol des. 77.5 περί sch Eustratius 95r-205r, 206r-266r Rec I copy of Marc. gr. Z. 316		e ₃
Vat. Barb. gr. 273 (<i>Diktyon</i> 64819)	Lorenzo Ciatti	16th	91r-122v	123r-157v	A	14	F	Y	conformal and systematic copy of Par. gr. 2450		
Vat. gr. 186 (<i>Diktyon</i> 66817)		13th	196r-243v	245r-286r	A	9a	N	N	sch [Michael of Ephesus]		F
Vat. gr. 195 (<i>Diktyon</i> 66826)		1342	1r-43r	43v-84v	A	N	B	Y	sch poor diagrams		
Vat. gr. 196 (<i>Diktyon</i> 66827)		14th	1r-15r	15r-30v	B	N	C	N	des. 147.1 εισαγωγῇ		E
Vat. gr. 197 (<i>Diktyon</i> 66828)		16th	1r-32r	32v-68v	D	N	N	N	inc. 3.3 ἀκίνητα correct quires order 1, 3, 4, 2, 5		
Vat. gr. 198 (<i>Diktyon</i> 66829)	Malachias	1360-70	2r-19v	21r-33v	A	1 13	[D]	N	marg Adj; 2r-19v, 19v-33v marg Rec I+ sch Dishypatos; sch Διππραχίου Soterichos splits <i>Ar</i> related to Marc. gr. Z. 318		

ms	copyist	date	Book I	Book II	I.23.15	Txt	Adj	gl	remarks	H	D
Vat. gr. 256 (<i>Diktyon</i> 66887)		14th in.	298r-316v	316v-336r	B	9	B	N	marg mix comm sch Michael of Ephesus related to Ambros. I 8 sup.		
Vat. gr. 1026 (<i>Diktyon</i> 67657)		14th	41r-63v	65v-84v	A	6	[A]	Y	Adj m rec same scholia as Matrit. 4678		
Vat. gr. 1040 (<i>Diktyon</i> 67671)		14th 15th	1r-20v	20v-42v	B	N	A	N	1r-4r restor. ad 19.12 ἴσα		
Vat. gr. 1051 (<i>Diktyon</i> 67682)		13th ex. – 14th in.	1r-20v	21r-40v	E	N	[A]	N	Adj 15th		
Vat. gr. 1411 (<i>Diktyon</i> 68042)		14th ex.	61r-76v	76v-92v	D	2 5.2	E	N	43r-60r, 93r-106r Rec II encl <i>Ar</i>		
Vat. gr. 2297 + Roma, Arch. S. Paolo 24C (<i>Diktyon</i> 68928 + 55907)		13th p.m.	3r-9v + 1r-54r	56r-111r	A	1 11a	[A]	N	1r-55r, 56r-111r Rec I sch Theodoros protocensor Adj marg Rec I		
Vat. gr. 2387 (<i>Diktyon</i> 69018)		13th p.m.	32r-48v	49r-67v	C	12	B	N	sch; cf. Hamburg, S-UB, philol. 88		
Vat. Ottob. gr. 310 (<i>Diktyon</i> 65553)		14th	124r-142r	142r-160r	B	N	A	N	copy of Vat. gr. 1040		V
Vindob. phil. gr. 62 (<i>Diktyon</i> 71176)		15th p.m.	1r-24r	24v-47v	A	N	A	N	no diagrams		
Vindob. phil. gr. 220 (<i>Diktyon</i> 71334)	Nikephoros	15th a.m.	21r-62r	63r-104v	A	14	F	Y	copy of Marc. gr. Z. 320		
Zeitz, Stiftsbibliothek 67 (Cizensis 67, <i>Diktyon</i> 72776)	John Argyropoulos	15th	36v-51v	51v-69r	D	2 5.2	E	N	3r-36r, 69v-96r Rec II encl <i>Ar</i> related to Vat. gr. 1411	C	C

MANUSCRIPTS OF NICOMACHUS, *INTRODUCTIO ARITHMETICA*, THAT DO NOT CONTAIN THE END OF BOOK I
OR THAT COULD NOT BE CHECKED (20)

ms	copyist	date	Book I	Book II	remarks
Athens, EBE 1238 (<i>Diktyon</i> 3534)		18th in.	1r-73v	74r-136r	marg Rec II ascr. Proclus ms. B D'Ooge possibly a copy of Athon. Meg. Laur. Θ.189
Athon. Meg. Laur. Θ.189 (Eustr. 1051) (<i>Diktyon</i> 28070)		17th	1r-	-68v	marg Rec II ascr. Proclus
Berol. Phillipps 1469 (<i>Diktyon</i> 9370)		14th	1r-4v		des 7.9 προδια[βεβαιωθέντα
Cantab. UL Gg.I.2 (gr. 1397) (<i>Diktyon</i> 12190)		15th	21r-24r		allegedly excerpts, but no segment of text mentioned in the catalogue is contained in <i>Ar</i>
Cantab. UL Kk.V.28 (gr. 2070) (<i>Diktyon</i> 12211)	Angelos Vergetios	15th m.	1r-	-88v	no scholia except on first three folios; n° 43 D'Ooge same spurious incipit as Par. gr. 2374
Cantab. UL Ll.V.4 (gr. 2202) (<i>Diktyon</i> 12224)		17th	225-253		marg Rec II; des mut Rec II Ι.πε.3 ἀρτιοπερίττου partial copy of Oxon. Lincoln gr. 33, as stated in the ms (cf catalogue)
Gelibolu, BK 6 (<i>Diktyon</i> 17158)		17th			no information available
Kozane, DB 42 (<i>Diktyon</i> 36946)		1705	97r-	-209v	further information not available
Istanbul, TSM G.İ.5 (<i>Diktyon</i> 33950)		14th	9r-35v	35v-63v	64r-184v Rec II
London, BL, Harley 6295 (<i>Diktyon</i> 39691)	Ioannes	15th p.m.		124v-126v	136.2 ὑπὸ τῶν ἄκρων usque ad finem
Marc. gr. 591 (coll. 1035) (<i>Diktyon</i> 70062)		16th in.	90r-106v	107r-119v	1-33.16 οἱ μὲν οὖν, 75.12 δηλονότι – 102.3 οἱ ι conformal copy of Vindob. phil. gr. 174

ms	copyist	date	Book I	Book II	remarks
Monac. gr. 238 (<i>Diktyon</i> 44685)	Boemundos	1348-66	1r-19r		des 50.2 πρὸς τῷ ὅλῳ stems from model of Roma, Arch. S. Paolo 24C + Vat. gr. 2297 copied in Southern Italy
Oxon. Bodl. Auct. T.4.4 (Misc. 242) (<i>Diktyon</i> 47190)		15th-16th	329r-343r	280r-287v 344r-v	1-37.7 εἴη, 75.12 δηλονότι – 99.15 μήκος conformal copy of Vindob. phil. gr. 174
Par. gr. 2375 (<i>Diktyon</i> 52007)	Constantinos Palaeocappas	16th	1-45		marg Rec I; no diagrams des 45.4 τὸ ἴσον ἐστίν partial and non exclusive copy of Par. gr. 2483
Scorial. T.I.11 (gr. 131) (<i>Diktyon</i> 15403)		16th	206-210		des 10.19 ὄν[των
Scorial. T.II.6 (gr. 145) (<i>Diktyon</i> 15417)	Michael Myrocephalites Antonios Eparchos	16th m.	1r-98r	103r-114v	I des 58.7 ἐπιπένθεκτος, II des. 147.1 εἰσαγωγῇ Book I copy of Ambros. B 77 sup. Book II comes from a different ms same desinit as Vat. gr. 196
Tyrnabos, DB 10 + 17 (<i>Diktyon</i> 64359 + 64366)		1675	115r-138v + 1r-44r	44r-109v	I.1-I.8.10 (17.5), I.11.3 (27.4) usque ad finem marg Rec II ascr. Proclus
Vat. gr. 199 (<i>Diktyon</i> 66830)		14th	1r, 2r-v		1-2.12 κόσμῳ, 1-6.9 ἀλήθειαν
Vat. gr. 1709 (<i>Diktyon</i> 68338)	copyist AG	15th a.m.	203r-210v		des. 28.15 τοῦ γ· καὶ conformal copy of Norimb. Cent. V. App. 36
Vindob. phil. gr. 174 (<i>Diktyon</i> 71288)		13th p.m.	133v-140v	141r-145v	correct folios order 133, 135-140, 134, 141-145 1-37.7 εἴη, 75.12 δηλονότι – 102.3 οἱ ι