

HOW TO SPELL THE GREEK ALPHABET LETTERS

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Abstract

A text contained in a 10th century manuscript and that provides a mathematical reason for spelling the Greek alphabet letters in a specific way is published and discussed.

Keywords: Greek alphabet letters, combinatorics, numerical notation, Byzantine mathematics

Resumen

El presente artículo presenta y analiza un texto, transmitido en un manuscrito del s. X, que ofrece el criterio matemático de una forma específica de deletrear las letras del alfabeto griego.

Metadata: Alfabeto griego, combinatoria, notación numérica, matemática bizantina

HOW TO SPELL THE GREEK ALPHABET LETTERS

FABIO ACERBI*

How to spell the Greek alphabet letters? In such a way that their sum as numeral letters yields exactly one myriad. This unexpected answer comes from the following text, to my knowledge first witnessed in the scrap of a 10th century manuscript preserved as Paris, Bibliothèque nationale de France, suppl. gr. 920 (22 ff., mm 150×110 on 19 lines; *Diktyon* 53604),¹ f. 1r-v,² and afterwards in monk Chariton's commonplace book Paris, Bibliothèque nationale de France, grec 1630, f. 101v (first half of 14th century; 270 ff., mm 158×115; *Diktyon* 51252).³ Let us first read the relevant portion of Par. suppl. gr. 920, f. 1r-v, accompanied by a translation:

* I am grateful to Stefano Valente for a suggestion, to Fabio Vendruscolo for keenly collaborating in reconstructing the corrupted sentence in Par. suppl. gr. 920, and to the anonymous referees for the critical remarks.

¹ Descriptions of the manuscript in *Catalogus Codicum Astrologorum Graecorum*, I-XII, Bruxelles 1898-1953 (henceforth CCAG), VIII.4, 89-92; Ch. Astruc – M-L. Concasty, *Bibliothèque Nationale. Catalogue des manuscrits grecs. Troisième Partie. Le Supplément Grec*, III, Paris 1960, 18-19. I shall edit elsewhere the chronological material that makes the bulk of the surviving folia (on f. 3r, the assumed current year is a.m. 6396 [= 887/8]; on f. 4v, it is a.m. 6400 [= 891/2]); for an orientation on such kind of texts, whose standard denomination “Easter Computi” is partly misleading, see O. Schissel, “Note sur un Catalogus Codicum Chronologorum Graecorum”, *Byz* 9 (1934), 269-295. Take notice of the fact that f. 1r opens with a very short excerpt from Herodianus' *Περὶ καθολικῆς προσφῳδίας* (*Grammatici Graeci*, I-IV, Lipsiae 1867-1910 [henceforth GG], III.1, 521.11-13).

² The first folio looks like a *feuille de garde* for the subsequent chronological text; their early codicological continuity is warranted by the Sicilian chronicle copied in the margins of ff. 1v-3r (date-range 827-982). Edition of the chronicle in P. Schreiner, *Die byzantinischen Kleinchroniken*, I-III (CFHB XII), Wien 1975-1979, I, 326-340 (nr. 45).

³ The copyist is identified in I. Pérez Martín, “Les *Kephalaia* de Chariton des Hodèges (Paris, BNF, gr. 1630)”, in P. van Deun – C. Macé (eds.), *Encyclopedic Trends in Byzantium? Proceedings of the International Conference held in Leuven, 6-8 May 2009* (Orientalia Lovaniensia Analecta 212), Leuven – Paris – Dudley (MA) 2011, 361-386, who also studied the initial, autograph *pinax*. Contents and bibliography in M. Tziatzi-Papagianni, *Die Sprüche der sieben Weisen. Zwei byzantinische Sammlungen*, Stuttgart – Leipzig 1994, 68-73; CCAG, VIII.3, 7-10. Our text, part of κεφάλαιον 18, is not included in the autograph *pinax*: Pérez Martín, “Les *Kephalaia*”, 369. A shorter version of *Text 1* is also in Par. gr. 2511, f. 62r (*Diktyon* 52143); this composite manuscript of end 14th century is partly a commonplace book.

[Text 1]

πῶς χρή μερίσαι τὰ κδ γράμματα εἰς τρία μέρη ισόψηφα;

A B Γ Z K C T Ω ,ατλγ

E H I M O P Y Ψ ,ατλγ

Δ Θ Λ Ν Ξ Π Φ Χ ,ατλγ

φέρει δὲ ὁ ψῆφος τῶν κδ στοιχείων ,γζοθ

[Text 2]

πῶς χρή κατ' οὐθοῦ ἐκφωνῆ⁴σθαι τὰ κδ | στοιχεῖα εἰς τὸν τῶν ἀριθμῶν;

ἄλφα βῆτα γάμμα δέλτα ,ασξη

εἰ ζῆτα ῆτα θῆτα γήνονται ζνη

ιώτα κάπα λάμδα μῦ γίνονται ,αψκθ

νῦ ξι οῦ πι γίνονται ,απ

ῥῶ σίγμα ταῦ υ γίνονται ,βσνε

φι χι ψι ῶ γίνονται ,βχλ

όμοῦ α'

[Text 1]

How must one divide the 24 letters in three isopsephic parts?

A B Γ Z K C T Ω 1333

E H I M O P Y Ψ 1333

Δ Θ Λ Ν Ξ Π Φ Χ 1333

and the “number” of the 24 letters amounts to 3999

[Text 2]

How must one correctly pronounce the 24 letters for <completing> the number of a myriad?⁵

ἄλφα βῆτα γάμμα δέλτα 1268

εἰ ζῆτα ῆτα θῆτα give 958

ιώτα κάπα λάμβδα μῦ give 1729

νῦ ξι οῦ πι give 1080

ῥῶ σίγμα ταῦ υ give 2255

⁴ εἰ marg. m.1

⁵ The sentence is locally corrupt and obviously incomplete; maybe what is missing is a final ψῆφον, which in the previous sentence is taken to be masculine, or perhaps we should correct to εἰς τὸν M^a ἀριθμόν (see Par. gr. 1630 below). The *vox nihili* οὐθοῦ might be a corruption of εὐθύ. Yet, since letters are invariant parts of speech, the specification κατ' εὐθύ “in the nominative” is pointless. Maybe one should intend “correctly”, but then the original syntagm could be something like κατ' ὀρθόν. The reading I have translated is πῶς χρή κατ' ὀρθόν ἐκφωνῆσαι τὰ κδ στοιχεῖα εἰς τὸν M^a ἀριθμόν;.

φĩ χĩ ψĩ ω̃ give 2630
together 1,0000⁶

Here is Par. gr. 1630, f. 101v, of which I only give the Greek text:

titulum euanidum

,γ̃ⲗⲣⲑ

A B Γ Z K C T Ω ὁμοῦ ,ατλγ

E H I M O P Y Ψ ὁμοῦ ,ατλγ

Δ Θ Λ Ν Ξ Π Φ Χ ὁμοῦ ,ατλγ

Εἰ δὲ ὀλόγραφα ταῦτα καθὼς ὑπὸ τῆς γλώττης ἐκφωνοῦνται. συμψηφίσεις, τὸν μύρια
συνίστασαι ἀριθμόν

Ἄλφα βῆτα γάμμα δέλτα ὁμοῦ ,ασξη

εἷ ζῆτα ἥτα θῆτα ὁμοῦ ⲗνη

ἰῶτα κάππα λάμ⁷δα μῦ ὁμοῦ ,αωθ

νῦ ξῖ οῦ πῖ ὁμοῦ ,απ

ῥῶ σίγμα ταῦ ῥ ὁμοῦ ,βσνε

φῖ χῖ ψῖ ῶ ὁμοῦ ,βχλ ὁμοῦ {α'}

Again in Par. gr. 1630, f. 102r, we read the following material,⁸ obviously related to what precedes:

Χριστὲ ὁ θεός : ,αφξε :: Σῶσον ἡμᾶς ,αφξε⁹ ::

Θεὸς : σπδ :: Ἅγιος : σπδ :: Ἀγαθός : σπδ ::

Ἄνθρωπος : ,ατι :: Τὸ πηλοῦ πλάσμα : ,ατι ::

Τὸ καλὸν φρόνημα : ,ατι :: Γεννᾶται ἵνα φάγη. πίη. ἀποθάνη : ,ατι ::

Θέβαιος φίλος , ,αρ :: Ἄρτι οὐδεὶς , ,αρ ::

Παῦλος , ψπα :: Σοφία , ψπα ::

Βασιλίσκος , ψμγ *:: Ἐμπρησμός . ψμγ ::

Κοσμάς , φλα :: λύρα , φλα ::

⁶ Of course, the sum total is 9920.

⁷ β s.l. m.2

⁸ This is printed in F. Boissonade, *Anecdota Graeca*, II, Paris 1830, 460-461, preceded by a text on isopsephy drawn again from Par. gr. 1630, f. 73r, l. 7 a.i. Boissonade thus correctly locates the source of the three pairs of isopsephic Homeric lines transcribed (with a mistake in the signature of the source manuscript) in *Auli Gellii Noctes Atticae* [...] *Recensione Antonii Thysii, J.C. & Jacobi Oisellii, J.C.*, Lugduni Batavorum 1666, 776-777, as a commentary on a well-known passage on isopsephy in Aulus Gellius, *Noctes Atticae* XIV.6.

⁹ expect. ambo ,αφξθ.

ἐξ οὗ καὶ στίχος : Κοσμάς ἀκούω. καὶ λύρα ψηφίζομαι¹⁰
καὶ ἄλλα τοιαῦτα μυρία ὁ ἐρευνῶν εὐρίσκει
Κὰν τοῖς Ὀμηρικοῖς στίχοις, εὐρίσκονται τοιαῦτα ἰσόψηφα
sequuntur
Iliadis H.264-265 ,γυφη m.2
Iliadis T.306-307 ,βωμ{δ} m.2
Odysseae ω.110-111 ,γυλς m.2¹¹
Σημεῖον συντελείας τοῦ παρόντος βίου

To understand what is happening in these texts, recall that in the standard Greek numerical notation there are 27 digits: the numerals from 1 to 999 are in fact denoted by the 24 current alphabet letters plus three additional ones, namely, ς (*digamma*),¹² ϣ (*koppa*), and ϝ (*sade* or *sampi*).¹³ These 27 letters are divided in three groups of nine, denoting in succession the numbers in the three lowest numerical orders: the units from 1 to 9, the tens from 10 to 90, and the hundreds from 100 to 900; the additional letters are assigned as follows: ς = 6, ϣ = 90, ϝ = 900.¹⁴ Thus, the 24 letters of the alphabet denote the units from 1 to 9 with the exclusion of 6 (summing to 39), the tens from 10 to 80

¹⁰ The line must be later than mid-8th century, since the Cosmas in question can only be Cosmas the Hymnographer, on which see K. Krumbacher, *Geschichte der Byzantinischen Litteratur*, 2nd ed., München 1897, 674-676.

¹¹ As a matter of fact, the first of these two lines has a ψηφος of 3102. It is difficult to explain a mistake of such an extent: the tradition of the Odyssey does not record relevant variant readings, but a difference of 334 can only be accounted for by a computation carried out on a different text.

¹² In Byzantine manuscripts and in modern editions this sign is always represented by the *sigma-tau* ligature known as *stigma*, almost identical to a form of *digamma* itself, and unfortunately very similar to final *sigma*.

¹³ Additional signs attached to the 27 digits were used to write numbers larger than 999. On this issue, see below and F. Acerbi – D. Manolova – I. Pérez Martín, “The Source of Nicholas Rhabdas’ *Letter to Khatzykes*: An Anonymous Arithmetical Treatise in Vat. Barb. gr. 4”, *JÖB* 68 (2018), 1-37.

¹⁴ M. N. Tod, “The Alphabetic Numeral System in Attica”, *ABSA* 45 (1950), 126-139; entirely second-hand, and historically and technically inadequate, S. Chrisomalis, *Numerical Notation. A Comparative History*, Cambridge 2010, 133-147. For a useful summary and a discussion of the most frequent copying mistakes affecting numerals, see F. Ronconi, *La traslitterazione dei testi greci* (Quaderni della Rivista di Bizantinistica 7), Spoleto 2003, 145-165. Number “zero” does not exist, but a sign for the “empty place” was currently used in tables and in the sexagesimal system: it is a small circle surmounted by a bar. For the form of this composite sign in papyri, see A. Jones, *Astronomical Papyri from Oxyrhynchus* (Memoirs of the American Philosophical Society 233), Philadelphia 1999, I, 61-62; in the oldest manuscripts of Ptolemy’s *Handy Tables*, see A. Tihon, Πτολεμαίου Πρόχειροι Κανόνες, *Les Tables Faciles de Ptolémée, volume 1a, Tables A1–A2* (Publications de l’Institut Orientaliste de Louvain 59a), Louvain-la-Neuve 2011, 58-59.

(summing to 360), and the hundreds from 100 to 800 (summing to 3600). On this basis, the $\psi\eta\phi\omicron\varsigma$ “number” of a string of alphabet letters is the sum of their values regarded as digits. For instance, the $\psi\eta\phi\omicron\varsigma$ of the word $\psi\eta\phi\omicron\varsigma$ is $700(\psi) + 8(\eta) + 500(\phi) + 70(\omicron) + 200(\varsigma) = 1478$. Two strings of letters having the same *psephos* are said to be isopsephic. Of course, using the alphabet letters as numerals gave rise to a rich literary production, on the border between numerology and *technopaignia*.¹⁵ Of the latter kind are also the remarkable numerical coincidences set out *exempli gratia* by Chariton on f. 102r of his commonplace book and that we have read above: tightly related syntagms that turn out to be isopsephic, a line amounting to a isopsephic riddle, three pairs of isopsephic consecutive Homeric lines.¹⁶

Our main texts, however, are arithmetical in character. As for *Text 1*, it is grounded on the fact that the sum of the numerical values of the 24 alphabet letters is 3999 (a number far more remarkable in our notation than in Greek notation);¹⁷ a tripartition of these letters is set out any of which yields exactly $\frac{1}{3}$ of 3999, namely, 1333. Actually, this is one of many possible tripartitions yielding the same numbers. To see this, note that units, tens, and hundreds only partly interfere in building the final sum. Thus, it is enough to select three independent, exclusive and exhaustive subsets of each numerical order each of which yields 13 (= 39:3), 120 (= 360:3), and 1200 (= 3600:3), respectively, in order to get three times 1333 by putting three such subsets coming from dif-

¹⁵ For the tradition of Greek arithmology, see F. E. Robbins, “The Tradition of Greek Arithmology”, *CPh* 16 (1921), 97-123. For Greek onomatomancy see P. Tannery, “Notice sur des fragments d’onomatomancie arithmétique”, *Notices et extraits des manuscrits de la Bibliothèque Nationale* 31 (1886), 231-260, repr. Id., *Mémoires scientifiques* IX, Toulouse – Paris 1929, 17-50, and O. Neugebauer – G. Saliba, “On Greek Numerology”, *Centaurus* 31 (1989), 189-206. For an overview of Byzantine arithmology, see F. Acerbi, “Arithmetic and Logistic, Geometry and Metrology, Harmonic Theory, Optics and Mechanics”, in S. Lazaris (ed.), *A Companion to Byzantine Science*, Leiden 2020, 105-159.

¹⁶ This can only be discovered by someone who computes the numerical value of every line of the Iliad and of the Odyssey. On this, and on isopsephy in general, see Ch. Luz, *Technopaignia. Formspiele in der griechischen Dichtung* (Mnemosyne. Supplements 324), Leiden 2010, 247-325 (in particular 251-253 and 301-302 for the texts in Par. gr. 1630, f. 102r, which the author only knows through the intermeditation of Boissonade); J. L. Hilton, “On Isopsephic Lines in Homer and Apollonius of Rhodes”, *CJ* 106 (2011), 385-394 (who relates second-hand, and thereby misleadingly, Boissonade’s findings); R. Ast – J. Lougovaya, “The Art of the Isopsephism in the Greco-Roman World”, in A. Jördens (ed.), *Ägyptische Magie und ihre Umwelt*, Wiesbaden 2015, 82-98. Recall that Apollonius’ treatise on numerical notation for large numbers computes the product of the numerical values of the letters of a line verse (and not its $\psi\eta\phi\omicron\varsigma$) as an example. Apollonius’ treatise is lost, but we read a detailed account in Pappus, *Collectio* II.

¹⁷ This is not an idle remark: read W. Brashear, “Βαινχωωωχ = 3663 – No Palindrome”, *ZPE* 78 (1989), 123-124.

ferent numerical orders together; one may then permute the subsets within the same numerical order. For example (I use our numerals), the 3-subset selections within the numerical orders

[4,9|5,8|1,2,3,7], [40,80|50,70|10,20,30,60], [400,800|500,700|100,200,300,600]

give rise to the three 1333-each partitions

{4,9|40,80|400,800}; {5,8|50,70|500,700}; {1,2,3,7|10,20,30,60|100,200,300,600}

but also, by interchanging the strings of numbers I have marked in the same way, to

{4,9|50,70|400,800}; {5,8|40,80|100,200,300,600}; {1,2,3,7|10,20,30,60|500,700}, etc.,

the partial sums of each of which are {13|120|1200}. The three subsets in the partition attested in Par. suppl. gr. 920, however, realize the required tripartition by means of the partial sums {13|20|1300}, {13|120|1200}, {13|220|1100}. Clearly, getting 3 as the final digit of each partial sum of the units and 1 as the initial digit of each partial sum of the thousands is mandatory, whereas the partition attested in *Text 1* shows that one can compensate the partial sum resulting from units with the one resulting from tens, and the one resulting from tens with the one resulting from hundreds. This fact makes the partition in *Text 1* less obvious than the one realized by using three times the partial sum {13|120|1200}, and suggests that systematic investigations might have been carried out. Such investigations might even have developed as far as a complete classification of all tripartitions of the 24 letters that give each time 1333 as partial sums.¹⁸

The outcome of strictly related, “second order” investigations (see below) is our *Text 2*. Spell in full letters the names of the 24 letters of the alphabet –that is, of the 24 digits– and compute the sum of the resulting strings of numerical letters: thus, ἄλφα gives 1 + 30 + 500 + 1 = 532,¹⁹ βῆτα, 311, etc. Then take the sum total, that is, the “number” of the numbers. Well, the result is simply startling: the sum total is exactly 1 myriad, namely, 10,000. Amazing. This is even more amazing since such a “number” of the numbers requires a further sign to be represented: either a trema superimposed to the standard digits, or a letter α put above a capital M. Unfortunately, the amazement is short-lived since there seems to be a bug. Look closely at the letters spelt out in the manuscripts: if

¹⁸ See the Appendix for this classification. It is clear that a sharply mathematically-minded author is required to develop the classification effectively.

¹⁹ By the way, the Computus in Par. suppl. gr. 920, f. 3r, provides the earliest occurrence I know of the mnemonic device that relates the standard Great Period of 532 years to the “number” of letter ἄλφα: ἡ δὲ περίοδος συνίσταται διὰ πεντακοσίων τριακονταδύο ἐτῶν, ἥγουν διὰ τοῦ ἀριθμοῦ τοῦ ἄλφα. We read a similar statement on lines 2-3 of the Computus, obviously related to the one in the Paris manuscript, published in F. P. Karthaler, “Die chronologischen Abhandlungen des Laurent. Gr. Plut. 57, Cod. 42. 154-162^v”, *BNJ* 10 (1933), 1-64, 4.

we exclude a mistake in Par. suppl. gr. 920 corrected in Par. gr. 1630,²⁰ all letters are spelt in standard ways²¹ apart from λάμδα, which is written λάμδα in both manuscripts (but note the correction *supra lineam* in Par. gr. 1630).

This is really annoying, since the unwelcome *beta* contributes just two units. Thus, someone carried out the calculation, found 1 myriad plus 2, desperately checked his own sums several times (as I have done myself haunted by the same feeling), and surrendered to the temptation of cheating. But did he cheat after all? For the calculator had in any case to select between alternative spellings of some letters corresponding to the same utterance, since for instance all letters ending in *iota* can also be spelt with a final diphtong *epsilon-iota*.²² The calculator also adopted a possible spelling of *lambda* –or, better for a philologist but worse for our calculator, of *labda*–²³ confirmed by some grammatical texts.²⁴ But then, conversely, this may be taken to be a *very good* reason to spell λάμδα, as the corrupt sentence opening *Text 2* suggests.

May we submit an informed guess as to the origin and the aim of our texts? First of all, there must have been an author, an excerptor (because of the texts' terseness), and a number of copyists (because of the mistakes they are affected by). This also means that

²⁰ Par. suppl. gr. 920 writes κάπα instead of κάππα, correctly yielding 1729 as the partial sum, but then the sum total is 9920, not 1 myriad; there is no way to restore the 80 units of the missing π. The letters are arranged in six 4-letter series in both manuscripts: in my opinion, this fact makes it certain that Par. gr. 1630 draws on the same source as Par. suppl. gr. 920, possibly being a copy of it. No surprise that Chariton amended the text.

²¹ A check of the entire *Grammatici Graeci* confirms this, including less obvious spellings such as the one of *epsilon* as the diphtong ει: cf. GG III.2, 432.35-433.6, an extract from Herodianus' Περί ὀρθογραφίας. For a complete check of the spellings in a writing outside the grammatical tradition, see Alypius, *Introductio musica*, in *Musici Scriptores Graeci*, ed. K. von Jan, Lipsiae 1895, 369-406. Of course, the forms we read in these texts may not be the original ones, but we are precisely interested in the way alphabetic letters were spelt by people writing in Greek: thus, the conventions adopted by the copyists are all the more relevant to the issue.

²² For the documentary record, see F. Th. Gignac, *A Grammar of the Greek Papyri of the Roman and Byzantine Periods. Volume I. Phonology* (Testi e Documenti per lo Studio dell'Antichità LV), Milano 1976, 189-191, and references therein. A first orientation, with bibliography, of the principal developments in the vowel system during the Roman period is provided by G. Horrocks, *Greek. A History of the Language and its Speakers*, 2nd ed., Chichester – Malden (MA) 2010, 160-170.

²³ See *LSJ*, s.v. λάβδα, and Plato, *Cratylus* 434C-435A, or Athenaeus, *Deipnosophistae* X, 453D = 485.15-25 Kaibel. The two forms are recoded in GG III.2, 541.18.

²⁴ These are a short tract included in the *scholia marciana* to Dionysius Thrax, GG I.3, in particular 322.7, and a similar tract edited in F. W. Sturz, *Etymologicum Graecae Linguae Gudianum*, Lipiae 1818, in particular 598 (I owe this clue to S. Valente). But recall that in early minuscule the difference between *beta* and *my* is just a matter of a leg: check this in Par. suppl. gr. 920, f. 1v.

our texts need not come from the same author. They do not need even if they could, for the results they report are strictly related albeit pertaining to different discursive orders:

- object language, namely, the alphabet letters are written as such: *Text 1* combines a couple of obvious facts –the sum total of the 24 alphabet letters taken as digits is 3999; this number is patently a multiple of 3– with a kind of combinatorial investigations that might have reached to non-trivial developments, since, as shown above, a partition less “natural” than others is set out in the text;
- metalanguage: in *Text 2*, the alphabet letters are written as they are spelt; the *psephos* of all of them, with some appropriate choice of spelling, turns out to be exactly 1 myriad; one might iterate the game in a number of ways: adding signs, for instance a suitable one for myriads, or writing each letter of the spelt out letters as it is spelt,²⁵ or introducing multiplication at some stage, etc.

Investigations mixing number theory and a modicum of *technopaignia* as those underlying *Text 1* might have been carried out any time from Apollonius to Late Antiquity.²⁶ Even if we adopt the most engaging mathematical scenario, the calculations involved in *Text 1* are well within reach in the said period. Thus, we should resist the temptation of assigning *Text 1* to the Hellenistic age on account of the facts that its marked mathematical character might seem naturally to locate it in the “Golden Age” of Greek mathematics or that other kinds of *technopaignia* date back to this period; after all, our sources do not attest to the diffusion of isopsephic literary elaborations before the 1st century. As for *Text 2*, whose connotation as a skillgame is obviously more prominent than in the case of *Text 1*, it might even be an *epanthêma* of longer elaborations of which *Text 1* is what remains.²⁷ *Text 2* appears in fact to presuppose acquaintance with grammatical elaborations and a wide diffusion of iotacism; the connection with grammar is strengthened by the presence, in Par. suppl. gr. 920, of the very short extract from Herodianus. Thus, *Text 2* is very likely a product of Late Antiquity or of the early Byzantine period. I shall refrain from guessing authors, but maybe calculating the *psephos* of the name of well-known scholarly commodities in the study of anonymous scientific texts might help divination.

²⁵ Thus, ἄλφα → ἀλφαλαμδαφιᾶλφα.

²⁶ For a reconstruction of ancient Greek combinatorics see F. Acerbi, “On the Shoulders of Hipparchus. A Reappraisal of Ancient Greek Combinatorics”, *AHES* 57 (2003), 465-502.

²⁷ On the correct meaning of *epanthêma* as “short complement to a comprehensive treatise”, whose best known specimen is Thymaridas’ general solution of a linear system of several unknowns, see P. Tannery, “Sur l’arithmétique pythagoricienne”, *Bulletin des Sciences Mathématiques*, 2^e série, 9 (1885), 69-88, repr. Id., *Mémoires scientifiques* II, Toulouse – Paris 1929, 179-201, 194-195.

Appendix. The partitions of the 24 letters

The series of numerical items that follow a “bold-face number + arrow” syntagm are the ways the bold-face number (“base number” henceforth) can be obtained as the sum of numbers within the sequences marked “units” and “tens & hundreds”.²⁸ For further reference, each of these numerical items is identified by an alphabet letter. A string of digits (“base string” henceforth) like 3 | 13 | 23 (resp. 10 | 13 | 13) represents the sums of the numbers in the subsets of a tripartition of the sequence marked “units” (resp. “tens & hundreds”; final zeros are eliminated in this case); as said, these sums sum up in their turn to 39 (resp. 36). The partitions associated to a base string are listed after it, with the following convention: a suitable item in the series of numbers yielding a base number included in the base string as sum is selected and set out by writing the alphabet letter that identify it; after the arrow, the items are listed that yield another base number included in the base string;²⁹ thus, “13 | 13 | 13 a → c, e, f; b → c, g” stands for the list of the partitions of 39: [4,9|5,8|1,2,3,7], [4,9|2,3,8|1,5,7], etc. (5 partitions in all); the selected base number is of course **13** in both instances. An indication “(**12** vs. **11**)” means that the base number **12** is selected first and that the other base number is **11**. Since the sum of the digits in any base string is given, only two base numbers must be selected. Thus, the list below sets out the *unordered* partitions of the relevant sequences.

Units 1 2 3 4 5 7 8 9

13 → ^a9,4 ^b9,3,1 ^c8,5 ^d8,4,1 ^e8,3,2 ^f7,5,1 ^g7,4,2 ^h5,4,3,1

13 | 13 | 13 a → c, e, f; b → c, g 5 part.

3 | 13 | 23 1,2 → a, c; 3 → a, c, d, f, g 7 part.

3 | 3 | 33 1,2 → 3 1 part.

Tens & hundreds 1 2 3 4 5 6 7 8

10 → ^a8,2 ^b7,3 ^c7,2,1 ^d6,4 ^e6,3,1 ^f5,4,1 ^g5,3,2 ^h4,3,2,1

11 → ^a8,3 ^b8,2,1 ^c7,4 ^d7,3,1 ^e6,5 ^f6,4,1 ^g6,3,2 ^h5,4,2 ⁱ5,3,2,1

12 → ^a8,4 ^b8,3,1 ^c7,5 ^d7,4,1 ^e7,3,2 ^f6,5,1 ^g6,4,2 ^h6,4,3,1 ⁱ5,4,3 ^l5,4,2,1

13 → ^a8,5 ^b8,4,1 ^c8,3,2 ^d7,6 ^e7,5,1 ^f7,4,2 ^g7,3,2,1 ^h6,5,2 ⁱ6,4,3 ^l6,4,2,1 ^m5,4,3,1

²⁸ This is called a “partition into distinct parts” of the base number. Here I list the numerical items in decreasing order.

²⁹ Either the first or the second selected base number is obvious and most of the times only one choice is available.

12 12 12	$a \rightarrow c, e, f, h; b \rightarrow c, g$	6 part.
11 12 13	$a \rightarrow d, e, g, i; b \rightarrow c, e, h; c \rightarrow a, b, f, g; d \rightarrow a, e, g; e \rightarrow e, f;$ $f \rightarrow a, c; g \rightarrow a, d; h \rightarrow c; i \rightarrow b; l \rightarrow a$ (12 vs. 11)	23 part.
10 13 13	$a \rightarrow d, f, g, i, l; b \rightarrow d, h; c \rightarrow d, e; d \rightarrow m; e \rightarrow i$ (13 vs. 13)	11 part.
1 2 33	$1 \rightarrow 2$	1 part.
1 3 32	$1 \rightarrow 3$	1 part.
1 12 23	$1 \rightarrow a, c, e, g, i$	5 part.
1 13 22	$1 \rightarrow a, c, d, f, h, i$	6 part.
2 3 31	$2 \rightarrow 3$	1 part.
2 11 23	$2 \rightarrow a, c, d, e, f$	5 part.
2 12 22	$2 \rightarrow a, b, c, d, f, i$	6 part.
2 13 21	$2 \rightarrow a, b, d, e, i, m$	6 part.
3 3 30	$1,2 \rightarrow 3$	1 part.
3 10 23	$1,2 \rightarrow b, d; 3 \rightarrow a, c, d, f$	6 part.
3 11 22	$1,2 \rightarrow a, c, e; 3 \rightarrow b, c, e, g, h$	8 part.
3 12 21	$1,2 \rightarrow a, c, l; 3 \rightarrow a, c, d, f, g, e$	9 part.
3 13 20	$1,2 \rightarrow a, d, i; 3 \rightarrow a, b, d, e, f, h, l$	10 part.

The following table sets out the flow chart that generates all tripartitions of the 24 Greek numeral letters; at a given step in the flow, asterisks replace numbers that are not constrained by the assignments of the problem, thereby giving rise to a combinatorial branching. Each threefold sequence represents a tripartition; a horizontal three-digit sequence represents a partition that sums up to 1333; each digit is the sum of the actual units, tens, and hundreds figuring in the partition. The number of partitions associated to each threefold sequence is calculated by multiplying the numbers of the partitions, as set out above, associated to the base strings; these can be read off, the zeros being possibly eliminated, as the three vertical strings of digits in the threefold sequence. Brackets enclose the combinatorial factors necessary to shift, as required, from unordered to ordered partitions within the base strings that feature at least two identical digits; division takes care of factoring out the contributions of identical rows within the threefold sequence. The partition in *Text 1* is shaded grey.

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